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ASPECTS OF THE QUARK MODEL FOR THE BARYONS

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ACADEMIC DISSERTATION

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Abstract

The chiral constituent quark model that describes baryons as systems of constituent quarks bound by interaction potentials, with mesons coupled to the quarks, has been employed in the study of electromagnetic and weak properties of light and strange baryons. Exchange current contributions to the baryon magnetic moments are implied by the flavor and spin dependent hyperfine quark-quark interaction of the model. It is shown that these, combined with contributions from a central confining interaction, largely compensate the relativistic corrections to the single-quark magnetic moment operator that otherwise would lead to underpredictions of the magnetic moments. By also taking into account relativistic corrections to the axial coupling constants in this model a unified description of the magnetic moments and the axial coupling constants of the baryons may be obtained.

The exchange charge density operators that are associated with a Fermi-invariant decomposition of quark-quark interactions have been constructed. By applying the chiral constituent quark model to calculations of the electromagnetic charge radii of the nucleons agreement with empirical data is achieved with reasonable values for the constituent quark charge radii.

Finally, the effects of an irreducible π-gluon exchange interaction between constituent quarks are studied. This interaction combined with the quark-quark interaction of the chiral constituent quark model and a weak gluon interaction has most of the features required to explain the hyperfine splittings of the nucleon and Δ hyperon spectra.

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1. Introduction

One of the challenges of subatomic physics during the early 1960's was how to explain the existence of the many hadrons, i.e. the strongly interacting particles that had been discovered during the previous decade. Among these were the so called hyperons, baryons with mass higher than the nucleon mass and which do not decay strongly to the nucleon (at that time the Δ, Λ, Σ and Ξ). The discovery of baryon (and meson) resonances both without and with "strangeness" was then made possible by the use of new types of detectors and many of the hyperon properties such as spin, parity, isospin etc. were determined experimentally. One way to understand the results was to group baryons together into multiplets by the use of symmetry arguments. This was done, independently, by e.g. Gell-Mann [1], Ne'eman [2], Speiser and Tarski [3] using the symmetry group SU(3). The success of this approach was confirmed by the discovery of the Ω^- hyperon [4], the existence of which had already been proposed by Gell-Mann [5] as a missing state in a baryon decuplet. The early quark model [6, 7] for baryons and mesons was the simplest model that realized this SU(3) symmetry. The baryons were suggested to be made out of three "quarks", while the mesons consisted of a "quark" and an "anti-quark", the quarks having non-integral charges and spin 1/2. Even though the quarks were not immediately interpreted as real physical quantities they were recognized as a useful theoretical tool for describing the hadrons.

By further combining quarks with different spin orientations baryons could be arranged in multiplets by the use of static SU(6) [8, 9] containing SU(3) for quarks of three different "flavors" and SU(2) for two different spin directions. This allowed properties such as the ratio of the neutron and proton magnetic moment [10, 11] to be predicted. The Δ_{33} resonance required that the quarks in a ground state decuplet formed completely symmetric wave functions in configuration space. Since the combined spin-flavor part of the wave function for these states also is symmetric the total wave function seemed to be symmetric, in conflict with Fermi-Dirac statistics. The suggestion of the existence of a new property for the quarks [12], called color, resolved this problem. The color part of the baryon wave function could now be made antisymmetric by using a combination of three colors, and be combined with the symmetric wave function part to yield a total wave function that is antisymmetric. It was also proposed [12] that there should be eight gauge vector fields associated with this new symmetry group called $SU(3)_C$. This observation would later be one of the fundamental ingredients of the theory of Quantum Chromodynamics (QCD) for strong interactions.

Using a simplistic assumption that the quark systems in hadrons could be treated as non-relativistic quantum mechanical systems where the quarks interact by forces that can be described by static potentials [13] the quark model together with a phenomenological, non-relativistic harmonic oscillator potential model resulted in a model [7, 14, 15, 16] that was able to explain many of the splittings in the baryon spectrum. In this model the quarks appear as "constituent" quarks bound by the non-relativistic potential, having effective masses of $\sim$ 300 MeV for light quarks (up and down) and $\sim$ 500 MeV for the strange quark (s), in contrast to the "current" quark masses ($<$ 10 MeV for u and d quarks and $\sim$ 100 MeV for the s quark) in the QCD Lagrangian.

In the 1970's a quantum theory of strong interactions, QCD, was developed (see e.g. [17, 18, 19]), in which the fundamental building blocks are spin 1/2 quarks with fractional electric charge and spin 1 gauge fields called gluons which interact with the quarks and among themselves. Both quarks and gluons carry color "charge" and the theory has $SU(3)_C$ symmetry. The strong interactions can be studied in QCD at high energies or large momentum transfer using perturbative techniques, since in this region the theory can be approximated as a weakly interacting field theory. This feature called asymptotic freedom [20, 21, 22] is supported by results in deep inelastic scattering experiments of electrons on protons where at high momentum transfer quarks inside nucleons seem to behave as free particles. For low-energy phenomena, on the other hand, with resonances and complicated interactions QCD cannot be solved perturbatively. In this region one then has to use models that include some of the main features of QCD. Since no free quarks have been observed, the models should be able to describe confinement as well as asymptotic freedom.

With the arrival of QCD some new features were added to the non-relativistic constituent quark model. An explicit dynamical quark model [23] within a QCD-inspired framework that included long-range flavor and spin independent confining forces and flavor SU(3) breaking via quark masses was developed. In this model also a short-range, spin dependent force coming from a non-relativistic reduction of one-gluon exchange between the constituent quarks was introduced, based on the concept of asymptotic freedom for quark-gluon interactions. By using some simplifying assumptions concerning the one-gluon exchange the model was then further developed (see e.g. [24, 25]).

Despite the success of the non-relativistic quark model (and later refined versions of the model) in explaining many features of the baryon spectrum and also in giving fairly good descriptions of observables such as magnetic moment by using baryon wave functions derived in the model, it has some shortcomings. One feature that is not present in the model is so called chiral symmetry. This symmetry is relevant in QCD in the limit of vanishing (current) quark masses since the QCD Lagrangian is then invariant under chiral transformations involving left- and right-handed quark fields separately. For quarks with small mass the theory has approximate chiral symmetry. Chiral symmetry would give rise to a parity doubling of states in the baryon spectrum, a feature which is seen in the high-energy region of the nucleon and Λ spectra. In the low-energy region, on the other hand, there are no parity partners, a fact which seems to indicate that the symmetry is spontaneously broken. The spontaneous breakdown of chiral symmetry, in turn, implies the existence of so called Goldstone bosons [26, 27], which in this case are believed to be the members of the light pseudoscalar meson octet (the π-, K- and η-mesons). It has been suggested [28] that these Goldstone boson fields can be treated as fundamental fields along with the quarks and gluons in the energy region between the confinement region ($\sim 100 - 300$ MeV) and the region of chiral symmetry restoration ($\sim$ 1 GeV) or, equivalently, for distances between that of spontaneous chiral symmetry breaking (0.2 - 0.3 fm) and the linear size of a baryon ($\sim$ 1 fm, corresponding to the inverse of the QCD confinement scale).

Recently a constituent quark model that seeks to include the concept of chiral symmetry was developed [29, 30, 31]. By including a chiral spin-flavor dependent interaction that is mediated by the pseudoscalar meson octet along with a central confining interaction and assuming the one-gluon exchange interaction to be negligible in the low-energy region it is possible to get good agreement with the empirical baryon spectrum, especially concerning the ordering of positive and negative parity states. The correct ordering of the parity states is possible due to the operator structure of the spin-flavor dependent interaction, which is not achieved with the spin-color dependent operator of one-gluon exchange.

Even though a satisfactory description of the baryon spectrum is achieved in this "chiral" constituent quark model the spectrum alone cannot determine the validity of the model or explain what dynamical mechanisms give rise to the chiral interaction. It is therefore important to test the model also by analyzing predictions for the electromagnetic (e.m.) properties of baryons, e.g. by studying the magnetic moment and the charge radius. By simultaneously studying predictions for parameters in weak interactions, e.g. the axial strength of semi-leptonic decays the model can further be scrutinized. The purpose of this thesis is thus to test different versions of the chiral constituent quark model on these grounds and to give some motivation for the use of the model.

Since the constituent quark mass is small compared to the baryon mass the e.m. and weak current operators will have significant relativistic corrections. It is convenient to include the relativistic corrections in momentum space by using quark Dirac spinors with the lower component being non-zero. When no relativistic corrections are included the quark model already gives quite good predictions for the magnetic moments of the baryons (see e.g. [10, 23, 32, 33]). The axial coupling constants are, however, overestimated [33]. Relativistic corrections will reduce the values of both the magnetic moment and the axial coupling constant. In the chiral constituent quark model the underestimation of the magnetic moments caused by the relativistic corrections can, however, be compensated for. The flavor dependent chiral interaction implies, due to the requirement of current conservation, that there are two-body exchange magnetic moment operators that will give contributions to the magnetic moment operator. (If the confining interaction further is viewed as a scalar exchange interaction this also will give contributions to the total magnetic moment.) The corresponding exchange current contributions to the axial coupling constants can be shown to be small, resulting in a (qualitatively) unified description of both observables in this model.

In elastic electron-proton scattering experiments the results show that the proton has a charge distribution different from that of a point particle [34, 35, 36, 37]. The neutron, on the other hand, has been studied in e.g. electron-deuteron scattering experiments and in scattering of slow neutrons off atomic electrons and has been found to also have a non-zero charge radius [38, 39, 40]. The charge radius of the nucleon is, along with the spectrum a direct manifestation of the internal nucleon structure. The study of the charge radius in the chiral constituent quark model is therefore of importance for testing the validity of the model.

The nucleon charge radius is experimentally determined from the slope of the corresponding electric Sachs form factor [41] at zero momentum transfer. Due to the definition of this form factor in terms of Dirac and Pauli form factors the charge radius will consist of two parts, one coming from the derivative of the Dirac part and the other coming from the anomalous magnetic moment of the nucleon. By combining the chiral constituent quark model results for the Dirac part with the empirical anomalous magnetic moment of the nucleon(s) the charge radius may thus be calculated. When taking into account both relativistic one-body corrections and exchange current corrections from the chiral and confining interactions good agreement with empirical data can be achieved, when assuming a reasonable value for the constituent quark charge radius.

In the chiral constituent quark model the one-gluon exchange interaction used in earlier constituent quark models is neglected. One reason for this is that the use of an exchange interaction of this form would give the wrong ordering of parity states in the baryon spectrum. There are further some indications from cooled lattice calculations [42] and from so called valence-QCD approximations [43, 44] that the quark-gluon coupling at small momentum transfer should be weak. This would then be part of the explanation for the empirically small spin-orbit splittings of the baryon spectrum. In the chiral constituent quark model good agreement with the empirical spectrum is achieved when using a flavor dependent spin-spin hyperfine interaction. In this case, however, an additional flavor dependent tensor interaction should be included for states other than the ground states, resulting in small spin-orbit splittings of low-lying negative parity resonance states having the wrong sign. These can, however, be compensated for by the tensor component of an irreducible π-gluon exchange interaction which necessarily appears if a one-gluon exchange interaction albeit very weak is combined with the the chiral interaction. By using a quasipotential framework [45] which allows the iterated one-gluon and one-pion exchange interactions to be extracted covariantly from the corresponding Bethe-Salpeter equation [46] the components of the irreducible π-gluon exchange interaction can be calculated. The central and spin-orbit components will be small and the tensor component of the same order but with opposite sign compared to the corresponding component of a one-pion quark-quark interaction, thus in effect cancelling the tensor component of the chiral quark-quark interaction. Finally, the spin-spin component will add to the corresponding spin-spin term in the chiral interaction, resulting in a strong attractive flavor dependent spin-spin interaction.

This thesis is divided into two parts, an introductory part and a part consisting of three published papers. The introductory part consists of seven chapters. Chapter 1 gives an introductory overview of the subject covered in the thesis and Chapter 2 introduces some central concepts of the quark model. In Chapter 3 there is some discussion on chiral symmetry and the chiral constituent quark model is introduced. Chapter 4 describes the calculation of some electromagnetic and weak observables (the magnetic moment, the axial coupling constant and the charge radius) for baryons when including one-body relativistic corrections to the relevant operators, while Chapter 5 deals with exchange current (two-body) corrections to the above observables. In Chapter 6 the irreducible π-gluon interaction is presented, and in Chapter 7 some conclusions are drawn. The second part of the thesis consists of Papers I, II and III. Paper I discusses one-body relativistic and two-body exchange current corrections to the magnetic moments and axial coupling constants of the light and strange baryons, Paper II contains a discussion on two-body exchange current corrections to the charge radii of the proton and the neutron, while Paper III presents calculations of the π-gluon exchange interaction.

2. Quarks in baryons

2.1. SU(3) classification of baryons

Baryons and their resonances can be organized in multiplets where the members of a multiplet have the same spin and parity quantum numbers. The ground state baryons form a spin-parity $\frac{1}{2}^+$ octet where the particles are labeled by the third component of their isospin, T_3, and their hypercharge, Y, which is the sum of the baryon number B and the strangeness number S. The relation between these quantum numbers is given by [47]

$$Q = T_3 + \frac{Y}{2} \,, \tag{2.1}$$

where Q is the electric charge of the baryon.

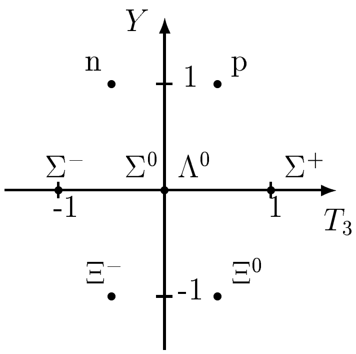


Figure 2.1: The baryon octet.

The lowest lying baryon resonances with spin-parity $\frac{3}{2}^+$, on the other hand, can be organized as a decuplet. Within these multiplets the masses of the particles are, if not equal, at least of the same order. The fact that there are deviations in mass in a multiplet indicates that the underlying symmetry is broken.

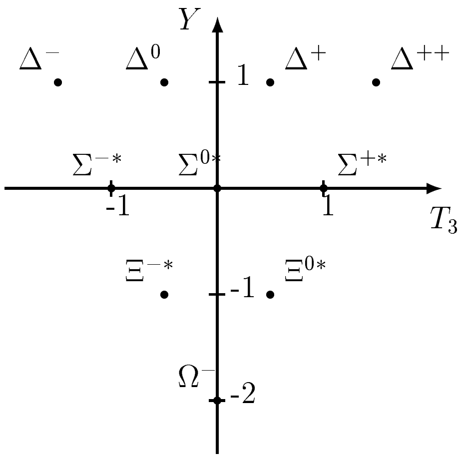


Figure 2.2: The baryon decuplet.

	u	d	s
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
T	$\frac{1}{2}$	$\frac{1}{2}$	0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0
S	0	0	-1
B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 2.1: Quantum numbers of the light quarks.

The existence of these multiplets can be explained by the quark model [6, 7] according to which the baryons may be described as bound systems of three quarks. The quarks are characterized by their flavor quantum number (up, down, strange, ...) and can come in three different "colors" (e.g. red, green, blue). The colors of the three-quark system have to be chosen so as to make the system "colorless", i.e. all of the three colors are present and combine to make a color singlet state. (Mesons, on the other hand, are built from a quark and an antiquark with color-anticolor combinations to make the system colorless.) If only the three lightest quarks (u, d, s) are considered the particles in the baryon octet and decuplet form irreducible representations of the symmetry group SU(3). The quantum numbers of the three light quarks are given in Table 2.1.

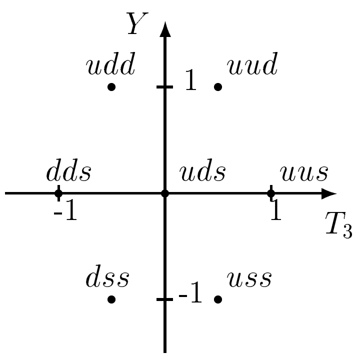


Figure 2.3: The quark content of the baryon octet.

$$
\begin{array}{l}
f_{123} = 1 \\
f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{637} = \frac{1}{2} \\
f_{458} = f_{678} = \frac{\sqrt{3}}{2} \\
d_{118} = d_{228} = d_{338} = -d_{888} = \frac{1}{\sqrt{3}} \\
d_{146} = d_{157} = d_{256} = d_{344} = d_{355} = \frac{1}{2} \\
d_{247} = d_{366} = d_{377} = -\frac{1}{2} \\
d_{448} = d_{558} = d_{668} = d_{778} = -\frac{1}{2\sqrt{3}}
\end{array}
$$

Table 2.2: Non-vanishing structure constants of SU(3).

The triplet $\phi \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ then transforms as $\phi' = U\phi$, where U is a 3×3 unitary matrix, belonging to the group SU(3). The matrix U can be written as

$$U = e^{\frac{1}{2}i\boldsymbol{\theta}\cdot\boldsymbol{\lambda}} \ , \tag{2.2}$$

where $\boldsymbol{\theta}$ is a constant vector and the components λ_i, $i = 1, \ldots,\ 8$, of the vector $\boldsymbol{\lambda}$ are the generators for the transformation, the so called SU(3) Gell-Mann matrices [48], defined by

$$
\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \quad
\lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} , \quad
\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} ,
$$

$$
\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} , \quad
\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} , \quad
\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} ,
$$

$$
\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} , \quad
\lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} . \tag{2.3}
$$

The Gell-Mann matrices satisfy the algebra

$$[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k \ , \tag{2.4a}$$

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\delta_{ij} + 2id_{ijk}\lambda_k \ , \tag{2.4b}$$

with the nonzero antisymmetric and symmetric structure constants f_{ijk} and d_{ijk}, respectively, given in Table 2.2. The quantum numbers characterizing the components of the baryon octet and decuplet can be identified as

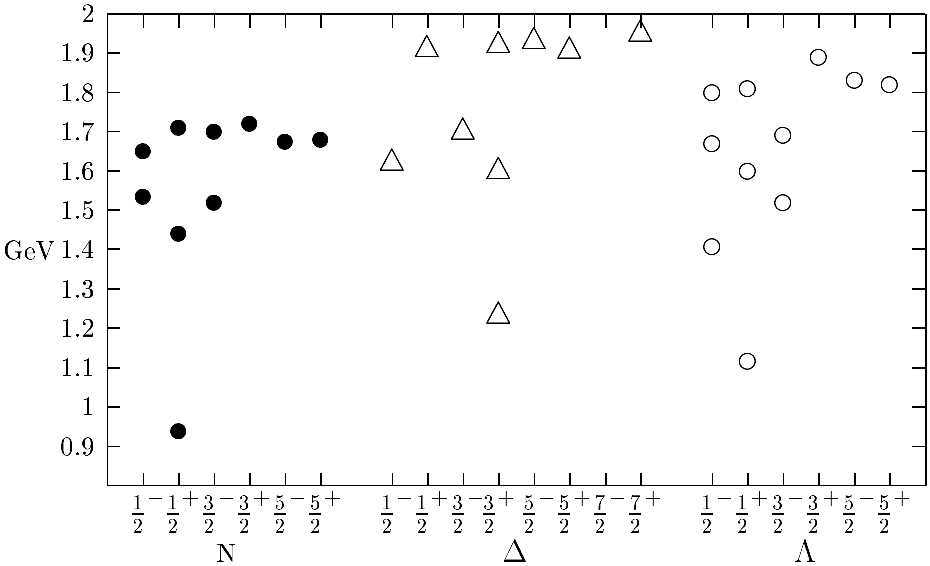


Figure 2.4: Spectra for N, Δ and Λ.

$$T_3 = \frac{1}{2}\lambda_3 \;, \qquad Y = \frac{1}{\sqrt{3}}\lambda_8 \;. \tag{2.5}$$

The dominant part of the quark-quark interaction is spin-independent. One could then describe the baryons as basic states of definite flavor (u, d, s) and spin (spin up or spin down), having $SU(3)_F \times SU(2)_S$ symmetry (if the mass difference caused by the mass of the s quark is neglected). This group is a subgroup of the group SU(6). One of the possible multiplets of SU(6) has the dimension $8 \times 2 + 10 \times 4 = 56$, and this multiplet is symmetric under interchange of any two quarks. The lowest octet and decuplet baryon states with the third spin component s_z being $1/2$, $-1/2$ and $3/2$, $1/2$, $-1/2$, $-3/2$, respectively, fill this representation. Higher resonance states, on the other hand, fill higher representations, e.g. one with the dimension 70. The mass degeneracy in the states is lifted by the breaking of the $SU(3)_F$ symmetry in the spin-dependent hyperfine interactions.

2.2. Quark dynamics

Free quarks have not been observed but seem to be confined to a small region of hadronic size. On the other hand, in processes that involve high momentum transfer, quarks seem to behave as free particles. This latter feature is called asymptotic freedom [20, 21]. A theory for quark dynamics should be able to explain both confinement and asymptotic freedom. QCD (Quantum Chromodynamics) [17, 18, 19] that describes strong interactions between quarks is a theory that seeks to include these features. The theory

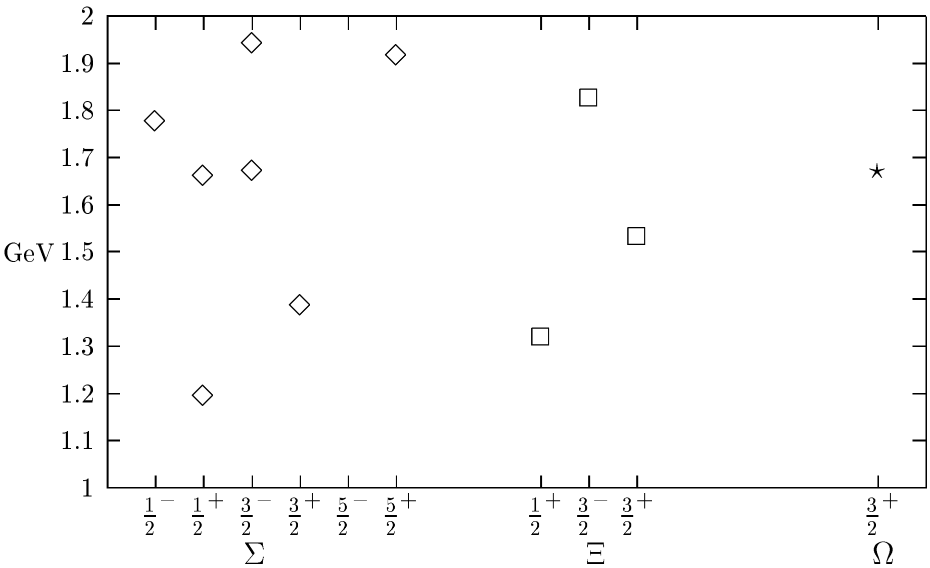


Figure 2.5: Spectra for Σ, Ξ and Ω.

introduces eight gluon fields which interact with the quarks. The quarks carry color"charge" and exchange massless colored gluons. The effective strong quark-gluon coupling constant α_S is momentum dependent and can be calculated perturbatively to first order as

$$\alpha_S(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{(2n_f - 33)}{12\pi} \ln\left(\frac{Q^2}{\mu^2}\right)} , \tag{2.6}$$

where μ is a scale parameter, $\alpha_S(\mu^2)$ is the coupling at μ^2, n_f is the number of flavors and Q^2 is the squared four-momentum transfer. If one chooses a scale parameter Λ so that

$$\Lambda^2 = \mu^2 e^{\frac{-12\pi}{(33-2n_f)\alpha_S(\mu^2)}} , \tag{2.7}$$

the above equation for α_S can then be written as

$$\alpha_S(Q^2) = \frac{12\pi}{(33 - 2n_f)} \cdot \frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} . \tag{2.8}$$

If $Q^2 \gg \Lambda^2$ the effective coupling is small and the interaction between the quarks and gluons can be described perturbatively. For $Q^2 \to \infty$ α_S vanishes and asymptotic freedom is realized. If, on the other hand, Q^2 is of the same order as Λ the perturbative calculation of α_S may no longer be valid.

To study confinement in the non-perturbative region one can employ different methods, e.g. study QCD on a lattice [49, 50] (so called lattice gauge theories), introduce bag models [51, 52], non-relativistic potential models such as the constituent quark model (see e.g. [23]) etc.. The confining interaction $V_{conf}(r)$ in the potential models should be a growing function of the interquark distance in order to describe confinement. One possibility is a harmonic confinement of the form $V_{conf}(r) = ar^2$, another is a linear confinement of the form $V_{conf}(r) = ar + b$, which is supported by lattice calculations of e.g. the heavy quark system [53]. In baryons the confining q-q interaction is assumed to be responsible for the gross features of the spectrum, the above examples resulting in a level ordering with alternating positive and negative parity states. The confining interaction should be complemented with some interaction of shorter range that describes the correct hyperfine structure of the baryon spectrum.

3. The chiral constituent quark model

3.1. Approximate chiral symmetry of QCD

The Dirac equation for a free spin 1/2 particle (e.g. a quark) has the form

$$(\gamma_\mu \partial_\mu + m)\psi(x) = 0 \ , \tag{3.1}$$

where m is the mass of the particle, γ_μ are the Dirac γ-matrices defined as $\gamma_\mu = (\boldsymbol{\gamma}, \gamma_4) = (\boldsymbol{\gamma}, i\gamma_0)$, with

$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & -i\boldsymbol{\sigma} \\ i\boldsymbol{\sigma} & 0 \end{pmatrix} \ , \tag{3.2a}$$

$$\gamma_4 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \ , \tag{3.2b}$$

and $\psi(x)$ is the wave function of the particle, called a spinor. For massless particles the Dirac equation reduces to $\gamma_\mu \partial_\mu \psi = 0$. If γ_5 is a combination of the γ-matrices defined as

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -\mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \ , \tag{3.3}$$

it is possible to construct another solution to the Dirac equation for massless fermions as $\gamma_5\psi$, since $\gamma_\mu\partial_\mu(\gamma_5\psi) = 0$. By combining the two solutions as

$$\psi_L = \frac{1}{2}(1+\gamma_5)\psi \ , \qquad \psi_R = \frac{1}{2}(1-\gamma_5)\psi \ , \tag{3.4}$$

one acquires solutions with definite chirality or handedness, i.e. left-handed and right-handed solutions, respectively. The Lagrangian for a massless non-interacting fermion can now be written as $\mathcal{L} = \mathcal{L}_L + \mathcal{L}_R$, where

$$\mathcal{L}_{L,R} = i\bar{\psi}_{L,R}\gamma_\mu\partial_\mu\psi_{L,R} \ . \tag{3.5}$$

The adjoint spinor $\bar{\psi}$ is defined as $\psi^\dagger\gamma_4$. The two Lagrangians $\mathcal{L}_{R,L}$ separately remain invariant under chiral phase transformations of the form

$$\psi_{L,R}(x) \to e^{-i\alpha_{L,R}}\psi_{L,R}(x) \ , \tag{3.6}$$

where $\alpha_{L,R}$ are constants that are real. This so called chiral symmetry is exact if the particles are massless. If, however, they have a small mass the symmetry is an approximate symmetry and the predicted consequences of the exact symmetry will be only approximately valid.

For massless u and d quarks the SU(2) chiral transformations can be written as $\psi_{L,R} \to e^{-i\boldsymbol{\theta}_{L,R}\cdot\boldsymbol{\tau}}\psi_{L,R}$, where $\psi_{L,R}$ are the chiral projections (3.4) of the doublet $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ (i.e. the left-handed and right-handed components of the fields are decoupled and have separate invariances), $\boldsymbol{\theta}_{L,R}$ are constant vectors and the components τ_i, $i = 1,\ 2,\ 3$, of the vector $\boldsymbol{\tau}$ are SU(2) Pauli matrices. If the strange quark is included as a massless particle the chiral SU(3) transformations will be of the form

$$\psi_{L,R} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}_{L,R} \rightarrow \psi' = e^{-i\boldsymbol{\theta}_{L,R}\cdot\boldsymbol{\lambda}}\psi_{L,R}\ , \tag{3.7}$$

where λ_a, $a = 1,\ \ldots\ ,8$ are SU(3) Gell-Mann matrices. The invariance under the chiral transformations (3.7) is called $\mathrm{SU(3)}_R \times \mathrm{SU(3)}_L$ invariance or, equivalently, $\mathrm{SU(3)}_V \times \mathrm{SU(3)}_A$ invariance if the transformation is redefined by using vector and axial-vector transformations.

Since the light quarks are not massless in QCD but have small (current) masses, the chiral symmetry is only approximate in this theory. The (current) quark masses of the u, d and s quarks can, however, be set to zero as a first approximation, and the actual deviations from zero treated as perturbations.

3.2. Spontaneously broken chiral symmetry

The approximate chiral invariance of the QCD Lagrangian is not reflected in the empirical baryon spectrum. For (approximate) chiral symmetry to be unbroken all baryon states would have approximately the same mass as another state of opposite parity but with the same spin, baryon number and strangeness. At least in the low-lying parts of the baryon spectrum this is not realized. A symmetry of this form is called "hidden", since the ground state of the theory does not have the symmetry of the Lagrangian.

The mechanism that causes a symmetry to be "hidden" is called spontaneous breaking of the symmetry. A special case of spontaneous symmetry breaking is dynamical symmetry breaking, with the symmetry breaking caused by the appearance of a vacuum expectation value of a composite operator and not of a fundamental field. According to Goldstone's theorem [26, 27] in a theory that has some continuous global symmetry of the Lagrangian which is not a symmetry of the ground state (the symmetry is spontaneously broken) there will be one or more massless spin-zero bosons (one for every independent broken symmetry). Since QCD is believed to have an approximate chiral symmetry that is spontaneously broken, as a consequence approximate Goldstone bosons will appear. These "pseudo-Goldstone" bosons are not massless, as in the case of exact chiral symmetry, but have a small mass [54], and are, according to a widely held view, the pseudoscalar octet mesons (π^+, π^-, π^0, K^+, K^-, K^0, $\bar{K}^0$, η). The pion mass can be related to the u and d (current) quark masses by [55]

$$m_\pi^2 = -\frac{(m_u + m_d)}{F_\pi^2} < 0|\bar{q}q|0> , \tag{3.8}$$

where F_π is the pion decay constant, with $F_\pi \approx 92$ MeV, and $< 0|\bar{q}q|0 >= \frac{1}{2} < 0|\bar{u}u+\bar{d}d|0 >$ is a vacuum expectation value called the quark condensate, which is nonzero in QCD. Its value has been determined to approximately $-(240 - 250 \text{ MeV})^3$ in lattice gauge calculations. The (approximate) chiral symmetry of QCD is thus dynamically broken and the quark condensate is an order parameter for the chiral symmetry breaking. Due to the breaking of the chiral symmetry the quarks will also acquire a momentum dependent dynamical mass which for small momenta can be linked to the constituent quark mass [56].

It has been suggested by Manohar and Georgi [28] that there should be two different scales in QCD with three flavors, one associated with the spontaneous breaking of the chiral symmetry, $\Lambda_{\chi SB} \simeq 1$ GeV, and another one, $\Lambda_{QCD} \simeq 100$ - 300 MeV, characterizing confinement. For distances smaller than $1/\Lambda_{\chi SB}$ the relevant degrees of freedom are current quarks and gluons. For distances beyond $1/\Lambda_{\chi SB} \simeq 0.2$ fm the valence current quarks will acquire their dynamical mass, i.e. they can be described as constituent quarks with masses of ~ 300 MeV for light quarks and ~ 500 MeV for strange quarks, and the Goldstone bosons associated with the spontaneous symmetry breaking will appear. On the other hand, $1/\Lambda_{QCD}$ is approximately the linear size of a baryon, beyond which the interaction should be described in terms of baryons and mesons. In the region between $\Lambda_{\chi SB}$ and Λ_{QCD} the effective Lagrangian would thus consist of gluon fields that are associated with the confining interaction between quarks and of constituent quarks and pseudoscalar meson fields (approximate Goldstone bosons).

3.3. The baryon wave function

3.3.1. Harmonic oscillator wave functions

The chiral constituent quark model [29, 30, 31] is similar to other so called constituent quark models based on SU(6) flavor-spin symmetry for baryons as far as the (unperturbed) baryon wave function is concerned. There are, however, considerable differences concerning the fine and hyperfine interactions used in this model compared to other constituent quark models (see e.g. [23, 24, 25, 57]), since the Goldstone bosons of spontaneously broken chiral symmetry are incorporated in the chiral constituent quark model along with constituent quarks.

The Hamiltonian that is used in the chiral constituent quark model consists of a spin-independent part H_{si} and a spin-dependent part H_{sd},

$$H = H_{si} + H_{sd} \ . \tag{3.9}$$

The spin-independent part can, for the non-relativistic version of the model [29], be written as

$$H_{si} = \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} V_{conf}(\mathbf{r}_{ij}) \ , \tag{3.10}$$

where V_{conf} is the confining interaction, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ is the separation between the constituent quarks and the constituent quarks are assumed to have the mass m . The harmonic oscillator approximation assumes that the quark-quark confining potential is of the form $\frac{1}{2}kr_{ij}^2$, which can be used to get zeroth-order eigenfunctions. If V_{conf} is not harmonic it is always possible to rewrite H_{si} as

$$\begin{aligned} H_{si} &= (\sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} \frac{1}{2}k\mathbf{r}_{ij}^2) + \sum_{i<j}(V_{conf}(\mathbf{r}_{ij}) - \frac{1}{2}k\mathbf{r}_{ij}^2) \\ &= H_0 + \sum_{i<j} U(\mathbf{r}_{ij}) \ , \end{aligned} \tag{3.11}$$

and treat $U(\mathbf{r}_{ij}) = V_{conf}(\mathbf{r}_{ij}) - \frac{1}{2}k\mathbf{r}_{ij}^2$ by perturbation theory. If the confining interaction is

$$V_{conf}(\mathbf{r}_{ij}) = \frac{1}{2}k\mathbf{r}_{ij}^2 + V_0 \ , \tag{3.12}$$

where V_0 is a constant, one would thus have $U(\mathbf{r}_{ij}) = V_0$.

It is now possible to solve exactly for the eigenvalues of the Hamiltonian

$$H_0 = \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} \frac{1}{2}k\mathbf{r}_{ij}^2 \ . \tag{3.13}$$

With a change of variables to

$$\begin{aligned} \mathbf{R} &= \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3}{3} \\ \mathbf{r} &= \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2) \\ \boldsymbol{\rho} &= \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3) \ , \end{aligned} \tag{3.14}$$

the Hamiltonian H_0 can be written as

$$H_0 = \frac{P_{cm}^2}{2(3m)} + (\frac{p_r^2}{2m} + \frac{3}{2}kr^2) + (\frac{p_\rho^2}{2m} + \frac{3}{2}k\rho^2) \ . \tag{3.15}$$

If the center-of-mass motion is subtracted H_0 describes two 3-dimensional degenerate harmonic oscillators, the lowest lying eigenstates of which are combined to give the ground state wave function

	$[3]_F$ (Symmetric states)		$[3]_F$ (Symmetric states)
Δ^{++}	uuu	Σ^{*0}	$\frac{1}{\sqrt{6}}(uds + dsu + sud$
Δ^{+}	$\frac{1}{\sqrt{3}}(udu + duu + uud)$		$+dus + sdu + usd)$
Δ^{0}	$\frac{1}{\sqrt{3}}(ddu + udd + dud)$	Σ^{*-}	$\frac{1}{\sqrt{3}}(dsd + sdd + dds)$
Δ^{-}	ddd	Ξ^{*0}	$\frac{1}{\sqrt{3}}(ssu + uss + sus)$
Σ^{*+}	$\frac{1}{\sqrt{3}}(usu + suu + uus)$	Ξ^{*-}	$\frac{1}{\sqrt{3}}(ssd + dss + sds)$
		Ω^{-}	sss

Table 3.1: Symmetric flavor parts of the baryon wave functions.

$$\psi_{00}(r,\rho) = (\frac{m\omega}{\pi})^{3/2} e^{-\frac{1}{2}m\omega(r^2+\rho^2)} \; . \tag{3.16}$$

Here the parameter ω is defined as $\sqrt{\frac{3k}{m}}$. The spatial part of the wave function for higher states can be constructed as combinations of products of wave functions for the two harmonic oscillators with appropriate quantum numbers. For a detailed classification of the harmonic wave functions for a three-quark system, see Refs. [58, 59].

3.3.2. Wave function notations

Generally, the spatial part of the baryon wave function can be written as $|\Psi_{NL}>=|N(\lambda\mu)L[f]_X(r)_X>$, where the notations of the translationally invariant shell model (TISM) [60] are used. The Elliott symbol $(\lambda\mu)$ [61] determines a harmonic oscillator SU(3) multiplet and L is the total orbital angular momentum. The Young pattern (diagram) $[f]_X$ indicates the spatial permutational symmetry of the state, so that [3] is a completely symmetric state, [21] is a state of mixed symmetry and [111] is a totally antisymmetric state. Finally, $(r)_X$ is the so called Yamanouchi symbol, which determines the basis vector of the irreducible representation $[f]_X$ of the permutation group S_3. For symmetric states $(r)_X = (111)$, and for antisymmetric states the corresponding symbol is (123). The mixed symmetric states can be described by the two basis vectors (112) (mixed symmetric $[21]^{M,S}$) and (121) (mixed antisymmetric $[21]^{M,A}$) (for a more detailed description of the wave functions in these notations, see Ref. [62]).

The flavor and spin parts, ϕ and χ, respectively, of the wave functions are also described by Young patterns (diagrams), where for the flavor part

	$[111]_F$ (Antisymmetric state)
$\Lambda(1405)$	$\frac{1}{\sqrt{6}}(uds + dsu + sud - dus - sdu - usd)$

Table 3.2: Antisymmetric flavor part of the baryon wave functions.

	$[21]_F^{M,S}$ (Mixed symmetric)	$[21]_F^{M,A}$ (Mixed antisymmetric)
p	$\frac{1}{\sqrt{6}}(2uud - udu - duu)$	$\frac{1}{\sqrt{2}}(udu - duu)$
n	$\frac{1}{\sqrt{6}}(dud + udd - 2ddu)$	$\frac{1}{\sqrt{2}}(udd - dud)$
Σ^+	$\frac{1}{\sqrt{6}}(2uus - usu - suu)$	$\frac{1}{\sqrt{2}}(usu - suu)$
Σ^0	$\frac{1}{\sqrt{12}}(2uds - dsu - sud$ $+2dus - sdu - usd)$	$\frac{1}{2}(usd + dsu - sdu - sud)$
Σ^-	$\frac{1}{\sqrt{6}}(2dds - dsd - sdd)$	$\frac{1}{\sqrt{2}}(dsd - sdd)$
Λ^0	$\frac{1}{2}(usd + sud - sdu - dsu)$	$\frac{1}{\sqrt{12}}(2uds - dsu - sud$ $-2dus + sdu + usd)$
Ξ^0	$\frac{1}{\sqrt{6}}(sus + uss - 2ssu)$	$\frac{1}{\sqrt{2}}(uss - sus)$
Ξ^-	$\frac{1}{\sqrt{6}}(sds + dss - 2ssd)$	$\frac{1}{\sqrt{2}}(dss - sds)$

Table 3.3: Mixed symmetry flavor parts of the baryon wave functions.

$[3]_{FS}$ (Symmetric states)	$[111]_{FS}$ (Antisymmetric states)
$[3]_F[3]_S$ $\frac{1}{\sqrt{2}}([21]_F^{M,A}[21]_S^{M,A} + [21]_F^{M,S}[21]_S^{M,S})$	$[111]_F[3]_S$ $\frac{1}{\sqrt{2}}([21]_F^{M,S}[21]_S^{M,A} - [21]_F^{M,A}[21]_S^{M,S})$
$[21]_{FS}^{M,S}$ (Mixed symmetric)	$[21]_{FS}^{M,A}$ (Mixed antisymmetric)
$[3]_F[21]_S^{M,S}$ $[21]_F^{M,S}[3]_S$ $\frac{1}{\sqrt{2}}([21]_F^{M,A}[21]_S^{M,A} - [21]_F^{M,S}[21]_S^{M,S})$ $[111]_F[21]_S^{M,S}$	$[3]_F[21]_S^{M,A}$ $[21]_F^{M,A}[3]_S$ $\frac{1}{\sqrt{2}}([21]_F^{M,S}[21]_S^{M,A} + [21]_F^{M,A}[21]_S^{M,S})$ $[111]_F[21]_S^{M,A}$

Table 3.4: Symmetry of combined flavor and spin wave functions.

the possible states are $[3]_F$, $[21]_F$ and $[111]_F$, while the spin part is either $[3]_S$ or $[21]_S$. The $[21]$ states can again be either mixed symmetric, $[21]^{M,S}$, or mixed antisymmetric, $[21]^{M,A}$. The flavor part of the wave function for different baryons is given in Tables 3.1 - 3.3. By combining the flavor and spin parts to a state $[f]_{FS}$ according to Table 3.4 the result will be symmetric, of mixed symmetry or antisymmetric. The combined flavor and spin wave function part $[f]_{FS}$ is then combined with the spatial part $[f]_X$ in a similar manner to get a totally symmetric state $[3]_{XFS}$. The total wave function should be antisymmetric when including the color part $[f]_C$. The color part is totally antisymmetric, i.e $[111]_C$, giving $[111]_{CXFS} = [111]_C \times [3]_{XFS}$. When calculating matrix elements and the energy of the states, the color part of the wave function can be factored out, since a possible color dependence of the confining interaction is the same for all quark pair states. The effective confining interaction can then be redefined to include this color factor.

For the ordering of the states it is sufficient to denote the states by the symbol

$$|\Psi_{\text{baryon}} >= |N(\lambda\mu)L[f]_X[f]_{FS}[f]_F[f]_S > \,, \tag{3.17}$$

so that the ground state nucleon is denoted by $0(00)0[3]_X[3]_{FS}[21]_F[21]_S$, and thus has mixed symmetry both in flavor and in spin, while e.g. the ground state for the Δ hyperon, with symmetric flavor and spin parts, is described by $0(00)0[3]_X[3]_{FS}[3]_F[3]_S$.

3.3.3. Baryon wave functions from semi-relativistic calculations

The spin-independent Hamiltonian (3.10) used above to derive the baryon wave function in the chiral constituent quark model is purely non-relativistic. For a semi-relativistic approach [30, 31] the kinetic energy term would be of the form $\sum_{i=1}^{3}\sqrt{\mathbf{p}_i^2+m_i^2}$. The wave function can then be derived e.g. by solving the so called Faddeev equations for a 3-body system [30, 63] or by using a stochastic variational method [31, 64, 65]. The resulting wave function ψ is symmetric only with respect to an interchange of quarks 1 and 2, but a symmetric wave function can be constructed as

$$\begin{aligned}\Psi^{SYM} &= N(1+\hat{P}_{12}+\hat{P}_{13}+\hat{P}_{23}+\hat{P}_{23}\hat{P}_{12}+\hat{P}_{13}\hat{P}_{12})\psi \\ &= N(1+\hat{P}_{13}+\hat{P}_{23})(1+\hat{P}_{12})\psi, \end{aligned} \tag{3.18}$$

where $\hat{P}_{ab}$ is an operator that interchanges the quarks a and b, and N is a normalization factor (ψ is not normalized to unity). Since ψ is symmetric with respect to $1 \leftrightarrow 2$, one has $\hat{P}_{13}=\hat{P}_{23}$ and $N(1+\hat{P}_{12})\psi = N'\psi$, resulting in

$$\Psi^{SYM} = N'(1+2\hat{P}_{23})\psi \ . \tag{3.19}$$

Normalization of Ψ^{SYM} to unity will give N'.

Wave functions of this type have been calculated with a linear confining interaction and a hyperfine interaction that is flavor dependent to get satisfactory spectra for the light and strange baryons [30, 31]. For the ground state of the nucleon the wave function in this model can be written as

$$\psi_N = (\phi\chi)_1\psi_1(r,\rho)+(\phi\chi)_2\psi_2(r,\rho) \ , \tag{3.20}$$

where $(\phi\chi)_i$ and $\psi_i(r,\rho)$, $i = 1,\ 2$ are the flavor-spin and spatial parts, respectively, of the wave function. The flavor-spin parts are defined as

$$(\phi\chi)_1 = [21]_F^A[21]_S^A \ , (\phi\chi)_2 = [21]_F^S[21]_S^S \ , \tag{3.21}$$

and the spatial parts are linear combinations of harmonic oscillator wave functions similar to Eq. (3.16),

$$\psi_i(r,\rho) = \sum_{k=1}^{n} C_{k,i}(a_{k,i}e^{-a_{k,i}r^2})(b_{k,i}e^{-b_{k,i}\rho^2}),\ i=1,2, \tag{3.22}$$

where $a_{k,i}$, $b_{k,i}$, and $C_{k,i}$ are constants.

3.4. Calculation of observables with the baryon wave function

When calculating one-body observables for the three-quark system one has to add all of the one-body parts,

$$< \Psi|\hat{O}_{tot}(\mathrm{one-body})|\Psi > = < \Psi|\hat{O}_1 + \hat{O}_2 + \hat{O}_3|\Psi > \ , \tag{3.23}$$

where $\hat{O}_i$ is a one-body operator and, correspondingly, for a two-body observable

$$< \Psi|\hat{O}_{tot}(\mathrm{two-body})|\Psi > = < \Psi|\hat{O}_{12} + \hat{O}_{21} + \hat{O}_{13} + \hat{O}_{31} + \hat{O}_{23} + \hat{O}_{32}|\Psi > \ , \tag{3.24}$$

where $\hat{O}_{ij}$ denotes a two-body operator. If the wave function used is symmetric with respect to interchange of any two quarks, the above equations reduce to

$$< \Psi|\hat{O}_{tot}(\mathrm{one-body})|\Psi > = 3 < \Psi|\hat{O}_1|\Psi > \ , \tag{3.25}$$

and

$$< \Psi|\hat{O}_{tot}(\mathrm{two-body})|\Psi > = 3 < \Psi|\hat{O}_{12} + \hat{O}_{21}|\Psi > = 6 < \Psi|\hat{O}_{12}|\Psi > \ . \tag{3.26}$$

If, on the other hand, the wave function is of the type mentioned in Section 3.3.3, i.e. it has to be symmetrized by hand, it is necessary to perform

$$\begin{aligned} < \Psi|\hat{O}_{tot}(\mathrm{one-body})|\Psi > &= < \Psi^{SYM}|\hat{O}_1 + \hat{O}_2 + \hat{O}_3|\Psi^{SYM} > \\ &= N' < \psi|(1 + 2\hat{P}_{23})(\hat{O}_1 + \hat{O}_2 + \hat{O}_3)|\Psi^{SYM} > \\ &= 3(N')^2 < \psi|(\hat{O}_1 + \hat{O}_2 + \hat{O}_3)(1 + 2\hat{P}_{23})|\psi > \ , \end{aligned} \tag{3.27}$$

and

$$\begin{aligned} < \Psi|\hat{O}_{tot}(\mathrm{two-body})|\Psi > &= < \Psi^{SYM}|\hat{O}_{12}+\hat{O}_{21}+\hat{O}_{13}+\hat{O}_{31}+\hat{O}_{23}+\hat{O}_{32}|\Psi^{SYM} > \\ &= N' < \psi|(1 + 2\hat{P}_{23})(\hat{O}_{12} + \hat{O}_{21} + \hat{O}_{13} + \hat{O}_{31} + \hat{O}_{23} + \hat{O}_{32})|\Psi^{SYM} > \\ &= 3(N')^2 < \psi|(\hat{O}_{12} + \hat{O}_{21} + \hat{O}_{13} + \hat{O}_{31} + \hat{O}_{23} + \hat{O}_{32})(1 + 2\hat{P}_{23})|\psi > \ . \end{aligned} \tag{3.28}$$

When the operators have a momentum dependence their expectation values can be calculated in the following way. The expectation value for $\hat{O}$ in configuration space is

$$< \Psi_{B'}|\hat{O}|\Psi_B > = < \phi'\chi'|I|\phi\chi > \ , \tag{3.29}$$

where I is an integral defined as

$$I = \int d^3r_1'd^3r_2'd^3r_3'd^3r_1d^3r_2d^3r_3\psi_f^*(\mathbf{r}_1', \mathbf{r}_2', \mathbf{r}_3')\hat{O}\psi_i(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \ . \tag{3.30}$$

Above $\psi_i(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ ($\psi_f^*(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{r}'_3)$) is the spatial part and $\phi\chi$ ($\phi'\chi'$) denotes the flavor-spin part of the initial (final) state wave function. The integral I can further be written as

$$I = \int \Pi_{i=1}^3 d^3 r'_i \Pi_{j=1}^3 d^3 r_j \psi_f^*(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{r}'_3) \frac{1}{(2\pi)^9} \int \Pi_{k=1}^3 d^3 p'_k \frac{1}{(2\pi)^9} \int \Pi_{l=1}^3 d^3 p_l$$
$$e^{i(\mathbf{r}'_1\cdot\mathbf{p}'_1 + \mathbf{r}'_2\cdot\mathbf{p}'_2 + \mathbf{r}'_3\cdot\mathbf{p}'_3)} \hat{O} e^{-i(\mathbf{r}_1\cdot\mathbf{p}_1 + \mathbf{r}_2\cdot\mathbf{p}_2 + \mathbf{r}_3\cdot\mathbf{p}_3)} \psi_i(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \ . \tag{3.31}$$

If the operator $\hat{O}$ above is a one-body operator in momentum space the impulse approximation will result in

$$\hat{O}_{(one-body)} = \hat{O}^{(1)}(2\pi)^6 \delta(\mathbf{p}'_2 - \mathbf{p}_2)\delta(\mathbf{p}'_3 - \mathbf{p}_3) + \hat{O}^{(2)}(2\pi)^6 \delta(\mathbf{p}'_1 - \mathbf{p}_1)\delta(\mathbf{p}'_3 - \mathbf{p}_3)$$
$$+\hat{O}^{(3)}(2\pi)^6 \delta(\mathbf{p}'_1 - \mathbf{p}_1)\delta(\mathbf{p}'_2 - \mathbf{p}_2) \ , \tag{3.32}$$

and for two-body operators one consequently has

$$\hat{O}_{(two-body)} = [\hat{O}^{(12)} + \hat{O}^{(21)}](2\pi)^3 \delta(\mathbf{p}'_3 - \mathbf{p}_3) + [\hat{O}^{(13)} + \hat{O}^{(31)}](2\pi)^3 \delta(\mathbf{p}'_2 - \mathbf{p}_2)$$
$$+[\hat{O}^{(23)} + \hat{O}^{(32)}](2\pi)^3 \delta(\mathbf{p}'_1 - \mathbf{p}_1) \ . \tag{3.33}$$

A change of variables in configuration space according to Eq. (3.14) combined with a corresponding change in the momentum variables yields

$$I = \int d^3R' d^3R d^3r' d^3r d^3\rho' d^3\rho \frac{1}{(2\pi)^{18}} \int d^3P'_{CM} d^3P_{CM} d^3p'_r d^3p_r d^3p'_\rho d^3p_\rho$$
$$\times \psi_f^*(\mathbf{R}', \mathbf{r}', \boldsymbol{\rho}') e^{i(\mathbf{R}'\cdot\mathbf{P}'_{CM} + \mathbf{r}'\cdot\mathbf{p}'_r + \boldsymbol{\rho}'\cdot\mathbf{p}'_\rho)} \hat{O} e^{-i(\mathbf{R}\cdot\mathbf{P}_{CM} + \mathbf{r}\cdot\mathbf{p}_r + \boldsymbol{\rho}\cdot\mathbf{p}_\rho)} \psi_i(\mathbf{R}, \mathbf{r}, \boldsymbol{\rho}) \ . \tag{3.34}$$

If one assumes that the initial and final spatial wave functions can be written as

$$\psi_i(\mathbf{R}, \mathbf{r}, \boldsymbol{\rho}) = \psi_i(\mathbf{r}, \boldsymbol{\rho}) e^{i\mathbf{P}_i\cdot\mathbf{R}} \ ,$$
$$\psi_f^*(\mathbf{R}', \mathbf{r}', \boldsymbol{\rho}') = \psi_f^*(\mathbf{r}', \boldsymbol{\rho}') e^{-i\mathbf{P}_f\cdot\mathbf{R}'} \ , \tag{3.35}$$

where $\mathbf{P}_i$ and $\mathbf{P}_f$ are the initial and final total momenta the integrals over R and R' collapse into δ functions, giving for the total integral

$$I = \int d^3r' d^3r d^3\rho' d^3\rho \frac{1}{(2\pi)^{12}} \int d^3P'_{CM} d^3P_{CM} d^3p'_r d^3p_r d^3p'_\rho d^3p_\rho$$
$$\times \psi_f^*(\mathbf{r}', \boldsymbol{\rho}') e^{i(\mathbf{r}'\cdot\mathbf{p}'_r + \boldsymbol{\rho}'\cdot\mathbf{p}'_\rho)} \hat{O} e^{-i(\mathbf{r}\cdot\mathbf{p}_r + \boldsymbol{\rho}\cdot\mathbf{p}_\rho)} \psi_i(\mathbf{r}, \boldsymbol{\rho})$$
$$\times \delta(\mathbf{P}_{CM} - \mathbf{P}_i)\delta(\mathbf{P}'_{CM} - \mathbf{P}_f) \ , \tag{3.36}$$

which further simplifies to

$$I = \frac{1}{(2\pi)^{12}} \int d^3p_r' d^3p_r d^3p_\rho' d^3p_\rho$$

$$\times \psi_f^*(\mathbf{p}_r', \mathbf{p}_\rho') \hat{O}(\mathbf{p}_r', \mathbf{p}_\rho', \mathbf{P}_f; \mathbf{p}_r, \mathbf{p}_\rho, \mathbf{P}_i) \psi_i(\mathbf{p}_r, \mathbf{p}_\rho) \ . \tag{3.37}$$

The wave function $\psi(\mathbf{p}_r, \mathbf{p}_\rho)$ is then the Fourier transform of $\psi(\mathbf{r}, \boldsymbol{\rho})$. In the harmonic oscillator model of Section 3.3.1. one has e.g. for the ground state baryon

$$\psi_{00}(\mathbf{p}_r, \mathbf{p}_\rho) = \left(\frac{4\pi}{m\omega}\right)^{3/2} e^{-\frac{1}{2m\omega}(p_r^2 + p_\rho^2)} \ . \tag{3.38}$$

The integral (3.37) can then be used in the original expression (3.29) for the expectation value of the observable.

3.5. The hyperfine interaction and the baryon spectrum

3.5.1. The chiral pseudoscalar interaction

As was shown in Section 3.3.1. the interaction between two constituent quarks can be described as consisting of a central spin-independent confining part, of e.g. harmonic form, and of fine and hyperfine parts that are spin-dependent. The gross features of the baryon spectrum can be described by the the confining interaction. When the spin- and flavor-independent Hamiltonian H_0 (3.15) (with the center-of-mass motion subtracted) is used and the quarks are assumed to have the same mass, the baryon spectra will be determined only by the orbital structure and the constituent quark mass. The ground state with $N = 0$ will have positive parity, the first excited states with $N = 1$ will have negative parity, the second excited states with $N = 2$ will have positive parity and so on. The ordering of the states would be in an alternating sequence of positive and negative parity states, a situation which is not realized in nature. By taking into account other interactions than the confining interaction between the quarks the ordering can then be altered in order to be consistent with empirical data.

Spin-orbit interactions (for states with orbital angular momentum L different from zero) cause a fine splitting of the spectrum while spin-spin interactions cause hyperfine splittings. If one assumes one-gluon exchange between constituent quarks i and j the interaction can be written as [23]

$$H_{OGE}(r_{ij}) = \frac{\alpha_S}{4} \boldsymbol{\lambda}_i^C \cdot \boldsymbol{\lambda}_j^C \left\{ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta(\mathbf{r}_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16}{3} \cdot \frac{\mathbf{s}_i \cdot \mathbf{s}_j}{m_i m_j} \right) \right.$$

$$-\frac{1}{2m_i m_j}\left(\frac{\mathbf{p}_i\cdot\mathbf{p}_j}{r_{ij}}+\frac{\mathbf{r}_{ij}\cdot(\mathbf{r}_{ij}\cdot\mathbf{p}_i)\mathbf{p}_j}{r_{ij}^3}\right)$$

$$-\frac{1}{2r_{ij}^3}\left(\frac{\mathbf{r}_{ij}\times\mathbf{p}_i\cdot\mathbf{s}_i}{m_i^2}-\frac{\mathbf{r}_{ij}\times\mathbf{p}_j\cdot\mathbf{s}_j}{m_j^2}\right.$$

$$\left.+\frac{1}{m_i m_j}\left[2\mathbf{r}_{ij}\times\mathbf{p}_i\cdot\mathbf{s}_j-2\mathbf{r}_{ij}\times\mathbf{p}_j\cdot\mathbf{s}_i-2\mathbf{s}_i\cdot\mathbf{s}_j+6\frac{(\mathbf{s}_i\cdot\mathbf{r}_{ij})(\mathbf{s}_j\cdot\mathbf{r}_{ij})}{r_{ij}^2}\right]\right)$$

$$+\ldots\Big\}\ , \tag{3.39}$$

where α_S is the strong coupling constant, the components of $\boldsymbol{\lambda}^C$ are color SU(3) matrices, $\mathbf{s}_i=\boldsymbol{\sigma}_i/2$ is the spin operator acting on the ith quark, $\mathbf{r}_{ij}$ is defined as $\mathbf{r}_i-\mathbf{r}_j$, and ... are relativistic corrections. The spin-spin dependent part of this interaction,

$$H_C\sim-\alpha_S\sum_{i<j}\frac{\pi}{6m_i m_j}\boldsymbol{\lambda}_i^C\cdot\boldsymbol{\lambda}_j^C\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\delta(\mathbf{r}_{ij})\ , \tag{3.40}$$

has often been used as the hyperfine interaction for the hyperfine splitting of the ground states in the baryon spectrum. It can explain some of the features of the fine structure in the baryon spectra, but has not been very successful in some other respects. One of the facts that has proven hard to explain is, as already mentioned, the different ordering of the positive and negative parity excited states for, on the one hand, the N and the Δ spectra and, on the other hand, the Λ hyperon spectrum. This problem cannot be overcome even if the radial behavior of (3.40) or the form for the confining interaction is changed due to the effects of the color operator structure $\boldsymbol{\lambda}_i^C\cdot\boldsymbol{\lambda}_j^C$ combined with the antisymmetry of the color part of the wave function for the baryon, giving $<\boldsymbol{\lambda}_i^C\cdot\boldsymbol{\lambda}_j^C>=-\frac{8}{3}$ for all baryons. Another problem arises concerning the large spin-orbit interaction that should accompany the color-magnetic interaction (3.40), but which empirically seems to be small.

A simpler explanation of the fine structure of the baryon spectra is achieved if one introduces a chiral pseudoscalar interaction, which in the $\mathrm{SU}(3)_F$ invariant limit has the form [29]

$$H_\chi\sim-\sum_{i<j}V(\mathbf{r}_{ij})\boldsymbol{\lambda}_i^F\cdot\boldsymbol{\lambda}_j^F\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\ , \tag{3.41}$$

where the components of $\boldsymbol{\lambda}^F$ are flavor SU(3) Gell-Mann matrices. In Eq. (3.41) $V(\mathbf{r}_{ij})$ is a potential which behaves as a Yukawa interaction at long range and has the behavior of some form of a smeared version of a δ function at short range. If the $\mathrm{SU}(3)_F$ symmetry is broken the term $V(\mathbf{r}_{ij})\boldsymbol{\lambda}_i^F\cdot\boldsymbol{\lambda}_j^F$ has the form

$$V(\mathbf{r}_{ij})\boldsymbol{\lambda}_i^F\cdot\boldsymbol{\lambda}_j^F=\sum_{a=1}^{3}V_\pi(r_{ij})\lambda_a^{(i)}\lambda_a^{(j)}+\sum_{a=4}^{7}V_K(r_{ij})\lambda_a^{(i)}\lambda_a^{(j)}+V_\eta(r_{ij})\lambda_8^{(i)}\lambda_8^{(j)}\ . \tag{3.42}$$

This form describes the different contributions from pion, kaon and η exchange, characterized by interactions between only light quarks, between a light and a strange quark and between any light and strange quark pair combination, respectively.

The reason for introducing this chiral pseudoscalar interaction is connected with the approximate chiral symmetry of the underlying QCD. The conclusion that this symmetry is not explicitly broken was, as already mentioned, drawn from observing the baryon spectra, noting that in the high energy part of the baryon spectra the baryon states have nearby parity partners. This feature is not seen in the low lying parts of the spectra, implying that the chiral symmetry of QCD instead seems to be spontaneously broken and realized in the hidden (Nambu-Goldstone) mode in this region. The spontaneous breaking of the chiral symmetry would then lead to the presence of the octet of pseudoscalar Goldstone bosons that couple directly to the constituent quarks.

3.5.2. Hyperfine splittings in the spectrum

The energy of different states of the spin and flavor independent Hamiltonian H_0 in Eq. (3.15) can be calculated as the sum of the energy of the two harmonic oscillators (denoted by r and ρ), resulting in $E_N = (3+N)\hbar\omega$, where N is the number of excitation quanta in the states. The quantum number N consists of the sum of the principal quantum numbers of the two harmonic oscillators, $N = N_r + N_\rho = (2n_r + l_r) + (2n_\rho + l_\rho)$, where $\mathbf{l}_r + \mathbf{l}_\rho$ equals the total (spatial) angular momentum $\mathbf{L}$. The states will be highly degenerate and additional quantum numbers are needed to characterize different states.

Next consider the spin-dependent part H_{sd} in (3.9) and treat it by first order perturbation theory. If the confining interaction V_{conf} is defined as in (3.12) the mass of the baryon states can then be expressed as [29]

$$M = \sum_{i=1}^{3} m_i + (3+N)\hbar\omega + 3V_0 + \delta M_{sd} \ , \tag{3.43}$$

where

$$\delta M_{sd} = < \Psi | H_{sd} | \Psi > \ . \tag{3.44}$$

In the wave function Ψ above the unperturbed oscillator wave function is used as an approximation and thus possible configuration mixing due to the spin-dependent hyperfine interaction is not taken into account.

To get consistency with the empirical baryon spectra it is possible to introduce the chiral pseudoscalar interaction (3.41), and thus the spin-dependent hyperfine splitting will be

$$\delta M_{sd} = \delta M_\chi =< \Psi| - \sum_{i<j} V(\mathbf{r}_{ij}) \boldsymbol{\lambda}_i^F \cdot \boldsymbol{\lambda}_j^F \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j |\Psi> \ . \qquad (3.45)$$

If the spatial part ($\psi_{NL} = \psi_{n_r l_r}(r)\psi_{n_\rho l_\rho}(\rho)$) and the combined flavor (ϕ) and spin (χ) parts of the wave function decouple the hyperfine splitting can be written in the form

$$\begin{aligned}\delta M_{sd} &= 3 < \psi_{n_r l_r}(\mathbf{r}_{12})|V_k(\mathbf{r}_{12})|\psi_{n_r l_r}(\mathbf{r}_{12}) >< \phi\chi| - \boldsymbol{\lambda}_1^F \cdot \boldsymbol{\lambda}_2^F \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 |\phi\chi > \\ &= 3P^k_{n_r l_r} < \phi\chi| - \boldsymbol{\lambda}_1^F \cdot \boldsymbol{\lambda}_2^F \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 |\phi\chi > , \qquad (3.46)\end{aligned}$$

where $k = \pi$, K, or η. The energy of a state is then expressed in terms of linear combinations of the radial matrix elements $P^k_{n_r l_r}$. By taking $V(\mathbf{r}_{ij})\boldsymbol{\lambda}_i^F \cdot \boldsymbol{\lambda}_j^F$ to be of the form suggested in Eq. (3.42) without parametrizing the form for V_π, V_K and V_η it is possible to achieve good agreement with empirical spectra if one matrix element $P^k_{n_r l_r}$, $k = \pi$, K, η, for each oscillator shell is extracted from the empirical mass splittings [29].

3.5.3. Parametrizations of the potential function

As was mentioned above good agreement with the baryon spectrum is obtained by extracting values for matrix elements from the empirical data without knowing the explicit form for the interaction potential $V(\mathbf{r}_{ij})$. The form of the potential can, however, also be parametrized to achieve agreement with the empirical spectrum. By comparison with a pseudoscalar meson exchange potential of the form

$$V(r) = \frac{-\mathrm{g}^2}{4\pi}\frac{1}{12m^2}\left\{4\pi\delta(r) - \mu^2\frac{e^{-\mu r}}{r}\right\} , \qquad (3.47)$$

which has a short range part in the form of a δ-function and a long range Yukawa part, a possible parametrization could be constructed as a phenomenologically determined function of the form [30]

$$V_\gamma(r_{ij}) = -\frac{\mathrm{g}^2}{4\pi}\cdot\frac{1}{12m_i m_j}\left\{\frac{4}{\sqrt{\pi}}\alpha^3 e^{-\alpha^2(r_{ij}-r_0)^2} - \mu_\gamma^2\frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}}H(r_{ij})\right\} , \qquad (3.48)$$

where the δ-function now has been "smeared" out over a range $1/\alpha$ ($\alpha = 2.91$ fm^{-1}) and $H(r)$ is a cut-off function for the Yukawa tail of the form $H(r) = \left\{1 - \frac{1}{1+e^{\beta(r-r_0)}}\right\}^5$, with $\beta = 20$ fm^{-1} and $r_0 = 0.43$ fm. The coupling constant in the model is $\frac{\mathrm{g}^2}{4\pi} = 0.67$, related to the nucleon-pion coupling constant $\mathrm{g}_{\pi N}$ as [29] $\mathrm{g} = \frac{3}{5}\frac{m_u}{m_N}\mathrm{g}_{\pi N}$, with $\frac{\mathrm{g}^2_{\pi N}}{4\pi} = 14.2$. The mass m is the constituent quark mass ($m_u = m_d = 340$ MeV, $m_s = 460$ MeV) and μ_γ the pseudoscalar meson mass, the index γ referring to π, K or η. Another possible parametrization would be [31]

$$V_\gamma(r_{ij}) = -\frac{\mathrm{g}^2}{4\pi} \cdot \frac{1}{12 m_i m_j} \left\{ \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}} - \mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} \right\} \ , \tag{3.49}$$

with Λ_γ being a parameter corresponding to π, K or η meson exchange defined as $\Lambda_\gamma = \Lambda_0 + \kappa\mu_\gamma$, with $\Lambda_0 = 2.87\,\mathrm{fm}^{-1}$ and $\kappa = 0.81$. Both parametrizations give good predictions for the spectra.

4. Electromagnetic and axial currents and observables

4.1. The electromagnetic current of a Dirac particle

The electromagnetic 4-current of a Dirac particle is defined as

$$J_\mu = i\bar{\psi}Q\gamma_\mu\psi \ , \tag{4.1}$$

where Q is the electric charge and ψ is the spinor of the particle. The Dirac γ-matrices are defined as in Section 3.1. For a free spin 1/2 particle the spinor can be taken as the positive energy plane wave solution of the Dirac equation (3.1), i.e.

$$\psi \sim u^{(s)}(\mathbf{p})e^{ip\cdot x} \ , \tag{4.2}$$

with $u^{(s)}$, $s = 1,\ 2$, defined as

$$u^{(s)}(\mathbf{p}) = \sqrt{\frac{E+m}{2E}}\begin{pmatrix} \chi^{(s)} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi^{(s)} \end{pmatrix} \ , \tag{4.3}$$

where $\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent spin up and spin down particles respectively. The definitions of the 4-vectors p_μ and x_μ are here $p_\mu = (\mathbf{p}, iE)$ and $x_\mu = (\mathbf{x}, it)$. If initial and final states are denoted by i and f Eq. (4.1) now takes the form

$$J_\mu \sim i\bar{u}_f^{(s)}Q\gamma_\mu u_i^{(s)}e^{-i(p_f-p_i)\cdot x} \ . \tag{4.4}$$

For light and strange quarks the charge operator Q can be defined as

$$Q = \frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{2}\lambda_3 \ . \tag{4.5}$$

By combining Q with the appropriate flavor state vector $\eta^{(f)}$, $f = u,\ d,\ s$, where $\eta^{(u)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\eta^{(d)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\eta^{(s)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, the electric charge of a quark is then given in units of e by $\eta^\dagger Q\eta$. Below the flavor state vector is included in the definition of $u(p)$. Assuming that the constituent quark can be treated as a point Dirac particle the matrix element of its electromagnetic current operator in the Heisenberg representation is

$$< p'|J_\mu(0)|p >= i\bar{u}(p'\ )[\frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{2}\lambda_3]\gamma_\mu u(p) \ , \tag{4.6}$$

where $|p>$ ($<p'|$) is the incoming (outgoing) particle state with momentum p (p'). Since $J_\mu = (\mathbf{J}, J_4) = (\mathbf{J}, iJ_0) = (\mathbf{J}, i\rho)$, the matrix elements of the spatial part of the current density operator and the electromagnetic charge density operator will be

$$< p'|\mathbf{J}(0)|p >= i\bar{u}(p'\ )[\frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{2}\lambda_3]\boldsymbol{\gamma}u(p)\ . \tag{4.7a}$$

and

$$< p'|\rho(0)|p >= \bar{u}(p'\ )[\frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{2}\lambda_3]\gamma_4 u(p)\ . \tag{4.7b}$$

Using Eq. (4.3) for the spinors the matrix element of the single-quark electromagnetic current density operator can now be written as

$$< p'|\mathbf{J}(0)|p >= -i\eta^\dagger\chi^\dagger[\frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{2}\lambda_3]$$

$$\sqrt{\frac{(E'+m)(E+m)}{4EE'}}[\frac{(i\mathbf{p}+\boldsymbol{\sigma}\times\mathbf{p})}{E+m} + \frac{(i\mathbf{p}'-\boldsymbol{\sigma}\times\mathbf{p}')}{E'+m}]\eta\chi\ . \tag{4.8}$$

Correspondingly, the matrix element of the single-quark electromagnetic charge density operator will take the form

$$< p'|\rho(0)|p >= \eta^\dagger\chi^\dagger[\frac{1}{2\sqrt{3}}\lambda_8 + \frac{1}{2}\lambda_3]$$

$$\sqrt{\frac{(E'+m)(E+m)}{4EE'}}[1 + \frac{\mathbf{p}'\cdot\mathbf{p}+i\boldsymbol{\sigma}\cdot\mathbf{p}'\times\mathbf{p}}{(E+m)(E'+m)}]\eta\chi\ . \tag{4.9}$$

4.2. The magnetic moment of a Dirac particle

The magnetic moment operator $\boldsymbol{\mu}$ due to an electric current distribution $\mathbf{J}(\mathbf{r})$ can be defined as

$$\boldsymbol{\mu} = \frac{1}{2}\int d^3r\mathbf{r}\times\mathbf{J}(\mathbf{r})\ . \tag{4.10}$$

By expressing $\mathbf{J}(\mathbf{r})$ in momentum space, using $\mathbf{J}(\mathbf{r}) = \int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\mathbf{J}(\mathbf{q})$, Eq. (4.10) can be written as

$$\boldsymbol{\mu} = \frac{1}{2}\int d^3q\frac{1}{(2\pi)^3}\int d^3re^{-i\mathbf{q}\cdot\mathbf{r}}(-i\boldsymbol{\nabla}_{\mathbf{q}}\times\mathbf{J}(\mathbf{q}))\ , \tag{4.11}$$

which, when noting that $\frac{1}{(2\pi)^3}\int d^3re^{-i\mathbf{q}\cdot\mathbf{r}} = \delta^{(3)}(\mathbf{q})$, can be rewritten as

$$\boldsymbol{\mu} = -\frac{i}{2}\lim_{\mathbf{q}\to 0}[\boldsymbol{\nabla}_{\mathbf{q}}\times\mathbf{J}(\mathbf{q})]\ , \tag{4.12}$$

To calculate the magnetic moment of a constituent quark the current operator in Eq. (4.8) should be used in Eq. (4.12). Since the constituent quarks have a small mass compared to e.g. the nucleon, there will be considerable relativistic corrections to the current operator. Their effect on the constituent quark magnetic moment can be calculated by introducing a velocity operator $\mathbf{v} = \frac{\mathbf{p}'+\mathbf{p}}{2m} \equiv \frac{\mathbf{P}}{m}$, where m is the constituent quark mass, and $\mathbf{p}$ and $\mathbf{p}'$ are the incoming and outgoing particle momenta respectively. The momentum transfer is $\mathbf{q} = \mathbf{p}' - \mathbf{p}$, and one has $\mathbf{p} = m\mathbf{v} - \frac{\mathbf{q}}{2}$, $\mathbf{p}' = m\mathbf{v} + \frac{\mathbf{q}}{2}$. Rewriting $< p'|\mathbf{J}(0)|p >$ in terms of the coordinates $\mathbf{v}$ and $\mathbf{q}$ now results in

$$
\begin{aligned}
< p'|\mathbf{J}(0)|p >= \frac{m}{2}\eta^\dagger\chi^\dagger Q \Big[f(E,E') \left(\mathbf{v} + \frac{i}{2m}\boldsymbol{\sigma}\times\mathbf{q}\right) \\
-g(E,E')\left(\frac{\mathbf{q}}{2m} + i\boldsymbol{\sigma}\times\mathbf{v}\right)\Big]\eta\chi\ ,
\end{aligned}
\tag{4.13}
$$

where

$$
f(E,E') = \frac{(E'+E+2m)}{\sqrt{EE'(E+m)(E'+m)}}\ ,\ \ g(E,E') = \frac{(E'-E)}{\sqrt{EE'(E+m)(E'+m)}}\ . \tag{4.14}
$$

The magnetic moment operator now takes the form

$$
\boldsymbol{\mu} = \frac{Q}{2m\sqrt{1+\mathbf{v}^2}}\left\{\boldsymbol{\sigma} - \frac{\mathbf{v}\times(\boldsymbol{\sigma}\times\mathbf{v})}{2\sqrt{1+\mathbf{v}^2}(1+\sqrt{1+\mathbf{v}^2})}\right\}\ . \tag{4.15}
$$

For particles that combine to ground state baryons the second term $\mathbf{v}\times(\boldsymbol{\sigma}\times\mathbf{v}) = \boldsymbol{\sigma}\mathbf{v}^2 - \mathbf{v}\boldsymbol{\sigma}\cdot\mathbf{v}$ can be reduced to $\frac{2}{3}\boldsymbol{\sigma}\mathbf{v}^2$, and one finally gets for the spin term in the magnetic moment operator of a constituent quark

$$
\boldsymbol{\mu}_{spin} = \frac{\boldsymbol{\mu}_{spin(non-rel)}}{\sqrt{1+\mathbf{v}^2}}\{1 - \frac{1}{3}\frac{\mathbf{v}^2}{\sqrt{1+\mathbf{v}^2}(1+\sqrt{1+\mathbf{v}^2})}\}\ , \tag{4.16}
$$

where $\boldsymbol{\mu}_{spin(non-rel)}$, defined as $\frac{1}{2m}Q\boldsymbol{\sigma}$, is the quark magnetic moment operator without relativistic corrections. By further noting that $\boldsymbol{\mu}_{spin(non-rel)}$ is equal to $\left(\frac{m_N}{m}\right)Q\boldsymbol{\sigma}$ in units of $\left(\frac{e}{2m_N}\right)$ the quark magnetic moment can finally, after some manipulations, be written in units of nuclear magnetons as

$$
\boldsymbol{\mu}_{spin} = \left(\frac{m_N}{m}\right)\frac{Q\boldsymbol{\sigma}}{\sqrt{1+\mathbf{v}^2}}\{1 - \frac{1}{3}(1 - \frac{1}{\sqrt{1+\mathbf{v}^2}})\}\ . \tag{4.17}
$$

4.3. The baryon magnetic moment

When using the (constituent) quark model in the impulse approximation, the baryon electromagnetic current operators are considered to be the sum of the current operators of the three constituent quarks, $J_\mu^{(tot)} = J_\mu^{(1)} + J_\mu^{(2)} + J_\mu^{(3)}$. If the wave function is symmetric when interchanging quarks, the expression for the electromagnetic current can be further simplified, since the current operators of the individual quarks are the same (if they all have the same mass), i.e., $J_\mu^{(tot)} = 3J_\mu^{(1)}$. When calculating the baryon magnetic moment with a wave function of this type one then only has to calculate $3 < \Psi'|\boldsymbol{\mu}_{spin}^{(1)}|\Psi >$, as shown in Eq. (3.25). The wave function is then separated into a flavor-spin part and a spatial part, and the baryon magnetic moment that includes relativistic corrections is calculated as

$$\mu_{baryon} = 3 < \phi'\chi'|\boldsymbol{\mu}_{spin(non-rel)}^{(1)}|\phi\chi >< \frac{1}{\sqrt{1+\mathbf{v}^2}}\{1-\frac{1}{3}(1-\frac{1}{\sqrt{1+\mathbf{v}}})\} > \ . \tag{4.18}$$

Here $\phi\chi$ is the combined flavor-spin wave function of the baryon, $\boldsymbol{\mu}_{spin(non-rel)}^{(1)}$ and $\mathbf{v}^2$ are now operators acting on the first quark, and the matrix element of the relativistic correction is calculated in momentum space as explained in Section 3.4. On the other hand, if the wave function is of the type mentioned in Section 3.3.3, i.e., it is symmetric only with respect to an interchange of quarks 1 and 2, the calculation of the baryon magnetic moment has to be made according to Eq. (3.27), where the one-body operator $\hat{O}_i$ now equals $\boldsymbol{\mu}_{spin}^{(i)}$, i.e.,

$$\mu_{baryon} = 3(N')^2 < \psi|(\boldsymbol{\mu}_{spin}^{(1)} + \boldsymbol{\mu}_{spin}^{(2)} + \boldsymbol{\mu}_{spin}^{(3)})(1+2\hat{P}_{23})|\psi > \ , \tag{4.19}$$

with $\hat{P}_{23}$ interchanging the coordinates of quarks 2 and 3.

In Paper I calculations of the magnetic moments without and with relativistic corrections have been done for the ground state baryons and the Δ^{++} and Ω^- hyperons. In the nucleon case the wave function used was the (exact) one derived in Ref. [30] implying the use of Eq. (4.19) while for the strange hyperons an oscillator model wave function approximation that in the nucleon case gives the same result as the exact wave function was used along with Eq. (4.18). The results are given in Table 1 of Paper I. The inclusion of the relativistic corrections reduces the impulse approximation results, which are already in quite good agreement with empirical data, by $20-30\%$ and thus considerably worsens the predictions. To get good agreement with data clearly some other contribution has to be added. When using the model of Ref. [31] with a harmonic oscillator wave function approximation that gives the same radius for the nucleon as the model in Ref. [31] the values for the proton and neutron magnetic moments with relativistic corrections are 1.36 and -0.91 n.m. [66], respectively, i.e. the relativistic corrections in this case reduce the impulse approximation results by $\sim 50\%$.

4.4. The axial current

When discussing weak semi-leptonic interactions between particles, e.g.

in β-decay of neutrons to protons a non-leptonic hadronic current is defined. In the Cabibbo theory [67] the hadronic weak current can be written as

$$J_\mu^h = \cos\theta_c J_\mu^h(\Delta S = 0) + \sin\theta_c J_\mu^h(|\Delta S| = 1) \ , \tag{4.20}$$

where θ_c is called the Cabibbo angle, $\Delta S = 0$ denotes processes where strangeness is conserved and $|\Delta S| = 1$ processes where strangeness changes by one unit. In the first type of processes electric charge and the third component of isospin change by one unit, $\Delta Q = \Delta T_3 = \pm 1$, while for the second case $\Delta Q = \Delta S = \pm 1$ and $\Delta T_3 = \pm\frac{1}{2}$. Both $J_\mu^h(\Delta S = 0)$ and $J_\mu^h(|\Delta S| = 1)$ consist of sums of vector and axial vector parts.

If the light and strange quarks are considered to be Dirac particles, it is possible to define eight quark axial currents as

$$A_{\mu a} = i\mathrm{g}_A^q \bar{\psi}_q \gamma_\mu \gamma_5 \frac{\lambda_a}{2} \psi_q \ , a = 1, \ldots 8, \tag{4.21}$$

where g_A^q is the axial current coupling of the quarks, λ_a are the eight flavor SU(3) Gell-Mann matrices, γ_5 is defined as in Eq. (3.3) and

$$\psi_q = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix} \ . \tag{4.22}$$

The value of g_A^q is 1 in the large color limit [68], and if the lowest $(1/N_C)$ corrections are included this value is reduced to $\mathrm{g}_A^q \approx 0.87$ [69]. Also one can define eight weak vector quark currents $V_{\mu a} \sim \gamma_\mu \frac{\lambda_a}{2}$, $a = 1, \ldots, 8$, and the hadronic weak current (4.20) can now be written as a combination of these components,

$$J_\mu^h = \cos\theta_c [V_{\mu(1+i2)} + A_{\mu(1+i2)}] + \sin\theta_c J_\mu^h [V_{\mu(4+i5)} + A_{\mu(4+i5)}] \ , \tag{4.23}$$

where $(a + ib)$ refers to a sum of the vector currents $V_{\mu a} + iV_{\mu b}$ or the axial vector currents $A_{\mu a} + iA_{\mu b}$, or, equivalently, to the combination $(\lambda_a + i\lambda_b)$ in the flavor matrices of the vector or axial vector current. Hence there is a strangeness non-changing axial current $A_{\mu(1+i2)}$, usually called $A_\mu^{(+)}$, defined by

$$\begin{aligned} A_{\mu(1+i2)} &= i\mathrm{g}_A^q \bar{\psi}_q \gamma_\mu \gamma_5 \frac{1}{2}(\lambda_1 + i\lambda_2)\psi_q \\ &= i\mathrm{g}_A^q \bar{\psi}_q \gamma_\mu \gamma_5 \tau_+ \psi_q \ , \end{aligned} \tag{4.24}$$

with

$$\tau_+ = \frac{1}{2}(\lambda_1 + i\lambda_2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{4.25}$$

which changes a d-quark to a u-quark and raises the electric charge by one unit e.g. in neutron β-decay. Similarly, one can define the strangeness changing axial current $A_{\mu(4+i5)}$ by

$$A_{\mu(4+i5)} = ig_A^q \bar{\psi}_q \gamma_\mu \gamma_5 v_+ \psi_q \ , \tag{4.26}$$

with

$$v_+ = \frac{1}{2}(\lambda_4 + i\lambda_5) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{4.27}$$

which changes an s-quark to a u-quark, as in β-decay of strange hyperons. Also in this process the electric charge is raised by one unit.

The adjoint operator $J_\mu^{h\dagger}$ of the hadronic weak current in Eq. (4.23) will, on the other hand, consist of vector and axial vector currents involving the combination $(\lambda_a - i\lambda_b)$ in their flavor parts. A strangeness non-changing axial current $A_{\mu(1-i2)}$, or $A_\mu^{(-)}$, will then have the same form as Eq. (4.24), but τ_+ will be replaced by τ_-,

$$\tau_- = \frac{1}{2}(\lambda_1 - i\lambda_2) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{4.28}$$

changing a u-quark to a d-quark and lowering the electric charge by one unit. The strangeness changing axial current $A_{\mu(4-i5)}$ is defined as in Eq. (4.26) with v_- replacing v_+,

$$v_- = \frac{1}{2}(\lambda_4 - i\lambda_5) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \ , \tag{4.29}$$

and this operator changes a u-quark to an s-quark and lowers the electric charge by one unit.

To see how relativistic corrections enter into the axial current operator the eight quark axial current operators can be written as

$$< p'|A_{\mu a}(0)|p >= ig_A^q \bar{u}(p')\gamma_\mu \gamma_5 \frac{\lambda_a}{2} u(p) \ , \ a = 1, \ \dots \ , \ 8, \tag{4.30}$$

where $u(p)$ is the Dirac spinor $\times$ flavor state vector for the quark. Since $A_{\mu a} = (\mathbf{A}_a, iA_0)$ one further gets for the matrix element of the vector part of the current

$$< p'|\mathbf{A}_a(0)|p >= ig_A^q \bar{u}(p')\boldsymbol{\gamma}\gamma_5 \frac{\lambda_a}{2} u(p) \tag{4.31a}$$

and for the matrix element of the time component

$$< p'|A_{0a}(0)|p >= g_A^q \bar{u}(p')\gamma_4 \gamma_5 \frac{\lambda_a}{2} u(p) \ . \tag{4.31b}$$

Using Eq. (4.3) one then has for the matrix element of the vector part of the axial current

$$< p'|\mathbf{A}_a(0)|p >= -\mathrm{g}_A^q \eta^\dagger \chi^\dagger \frac{\lambda_a}{2} \sqrt{\frac{(E'+m)(E+m)}{4E'E}}$$

$$\{\boldsymbol{\sigma} + \frac{[\boldsymbol{\sigma}\cdot\mathbf{p}'\mathbf{p} + \boldsymbol{\sigma}\cdot\mathbf{p}\mathbf{p}' - \boldsymbol{\sigma}\mathbf{p}'\cdot\mathbf{p} - i\mathbf{p}'\times\mathbf{p}]}{(E'+m)(E+m)}\}\eta\chi \ . \qquad (4.32)$$

By substituting $\mathbf{p} = m\mathbf{v} - \frac{\mathbf{q}}{2}$ and $\mathbf{p}' = m\mathbf{v} + \frac{\mathbf{q}}{2}$ the relativistic corrections can be calculated and the above reduces to

$$< p'|\mathbf{A}_a(0)|p >= -\mathrm{g}_A^q \eta^\dagger \chi^\dagger \frac{\lambda_a}{2} \boldsymbol{\sigma}\{1 - \frac{2}{3}(1 - \frac{1}{\sqrt{1+\mathbf{v}^2}})\}\eta\chi \ . \qquad (4.33)$$

In the non-relativistic limit the matrix element of the vector part of the quark axial current operator is then

$$< p'|\mathbf{A}_a(0)|p >= -\mathrm{g}_A^q \eta^\dagger \chi^\dagger \frac{\lambda_a}{2} \boldsymbol{\sigma}\eta\chi \ . \qquad (4.34)$$

In the above derivation the lower index a on the axial current operator and the flavor matrix could also, of course, be substituted for $(a \pm ib)$ to describe strangeness non-changing $(a = 1,\ b = 2)$ and strangeness changing $(a = 4,\ b = 5)$ processes.

4.5. The axial coupling constant of the baryons

For baryons the weak axial current operator can be written in the general form

$$< p'|_{B'} A_\mu(0)|p >_B = i\bar{u}(\mathbf{p}')_{S'}[\gamma_\mu \mathrm{g}_A(q^2) + i(p'-p)_\mu \mathrm{g}_P(q^2)]\gamma_5 u(\mathbf{p})_S \ , \quad (4.35)$$

where B (B') is the incoming (outgoing) baryon with Dirac spinor $u(\mathbf{p})_S$ $(u(\mathbf{p}')_{S'})$ and $\mathrm{g}_A(q^2)$ and $\mathrm{g}_P(q^2)$ are form factors which are determined experimentally. If the four-momentum transfer $(p'-p)_\mu$ is small, the second term in Eq. (4.35) can be neglected. In the case of e.g. neutron β-decay the relevant matrix element would be taken between an incoming neutron and an outgoing proton with the axial current for a strangeness non-changing process of the form $A_{\mu(1+i2)}$. For β-decays of hyperons the strangeness changing axial current $A_{\mu(4+i5)}$ would be relevant.

The form factor $\mathrm{g}_A(q^2)$ can be parametrized as

$$\mathrm{g}_A(q^2) = aF(q^2) + bD(q^2) \ , \qquad (4.36)$$

where the constants a and b are generalized Clebsch-Gordan coefficients [70]. In experiments with neutrino scattering on deuterons [71, 72] the axial form factor for the process $n \to p$ has been measured and the data fitted by the form $g_A^{n\to p}(q^2) = \frac{1.23}{(1+q^2/M_A^2)^2}$, $q^2 = (\mathbf{p}' - \mathbf{p})^2 - (E' - E)^2$, giving for the axial coupling constant $g_A = g_A^{n\to p}(0) = 1.23$. Estimates of $g_A^{n\to p}(0)$ have also been made from β-decay measurements, giving the value 1.259 ± 0.017 [73] for $g_A(0)/g_V(0)$, where g_V is a form factor associated with the vector part of the weak hadronic current for nucleons, the value of which is 1 for zero four-momentum transfer. If the momentum transfer is small it is possible to approximate the functions $F(q^2)$ and $D(q^2)$ in Eq. (4.36) with their values at $q^2 = 0$. One fit to experimental data [74] which gives $F = F(0) = 0.451 \pm 0.019$ and $D = D(0) = 0.777 \pm 0.021$ yields $g_A(0) = 1.228$ for the nucleon .

If F and D are calculated theoretically in the static quark model, assuming that the baryon axial current operator is the sum of three static quark axial current operators $\mathbf{A}_{(a+ib)non-rel}$ derived from Eqs. (4.24) or (4.26) (the impulse approximation) with $g_A^q = 1$ one yields $F = \frac{2}{3}$ and $D = 1$. The corresponding value for $g_A(0)$ will then be overestimated compared to the experimental value. However, by including relativistic corrections in the single quark operator as shown in Eq. (4.33) and calculating $< \mathbf{A}(0)_{(a+ib)} >_{baryon}$ in the same manner as the baryon magnetic moment was calculated in Section 4.3 the theoretical value of $g_A(0)$ can be reduced. Also by including corrections to the constituent quark axial coupling constant g_A^q as mentioned in the beginning of Section 4.4 one can further reduce the theoretical value of $g_A(0)$ for the baryons, and thus yield values that are in relatively good agreement with the empirical ones. Numerical calculations of the relativistic correction to the axial coupling constants of the baryon octet have been performed in the chiral constituent quark model of Ref. [30] where for the strange hyperons a harmonic wave function approximation was used, the result of which is consistent with the result for the exact wave function of Ref. [30] when applied to the nucleon. The calculated values are given in Table 2 of Paper I and show a reduction of $\sim 10 - 20\%$ from the static quark model values. Thus the predictions for the axial coupling constants when including relativistic corrections are close to the empirical ones.

4.6. The charge radius

In Section 4.1 the electromagnetic current of a Dirac (point) particle was defined. When studying baryons this simple form for the current can no longer be used, since the internal structure of the baryon has to be taken into account. Phenomenologically, the matrix element of the electromagnetic current operator for a baryon can be defined as

$$< p'|_B J_\mu^{em}(0)|p >_B = i\bar{u}(\mathbf{p}')_{S'}[\gamma_\mu F_1(q^2) - \frac{\sigma_{\mu\nu}q_\nu}{2M}F_2(q^2)]u(\mathbf{p})_S \ , \tag{4.37}$$

where $u(\mathbf{p})$ is the Dirac spinor of the baryon B, p (p') and S (S') denote the four-momentum and the spin structure of the incoming (outgoing) baryon, and M is the baryon mass. The tensor $\sigma_{\mu\nu}$ is defined here as $\frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, and $q_\nu = (\mathbf{q}, iq_0)$ is the four-momentum transfer, $q = p' - p$. The functions F_1 and F_2 are called the Dirac and Pauli form factors, respectively, and they describe the finite electromagnetic structure of the baryon. The form factors are defined so that $F_1(0) = Q$ and $F_2(0) = \kappa$, where Q is the electric charge and κ the anomalous magnetic moment of the baryon. Since $J_\mu = (\mathbf{J}, i\rho)$, the matrix element of the one-body baryon charge density operator can be defined as

$$\rho^{em,B} =< p'|_B \rho^{em}(0)|p >_B = \bar{u}(\mathbf{p}')_{S'}[\gamma_4 F_1(q^2) - \frac{\sigma_{4\nu}q_\nu}{2m}F_2(q^2)]u(\mathbf{p})_S \ . \tag{4.38}$$

By denoting

$$u(\mathbf{p})_S = \sqrt{\frac{E+M}{2E}} \begin{pmatrix} \chi_B^{(S)} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\chi_B^{(S)} \end{pmatrix} \ , \tag{4.39}$$

where E and M are the baryon energy and mass, and $\chi_B^{(S)}$ represents the baryon spin state, the charge density operator matrix element can be cast in the form

$$\rho^{em,B} = \chi_B^{(S')\dagger}\sqrt{\frac{(E'+M)(E+M)}{4EE'}}\Big\{F_1[1 + \frac{\mathbf{p}'\cdot\mathbf{p} + i\boldsymbol{\sigma}\cdot\mathbf{p}'\times\mathbf{p}}{(E+M)(E'+M)}]$$

$$+\frac{F_2}{2M}[\frac{\mathbf{p}'\cdot\mathbf{q} + i\boldsymbol{\sigma}\cdot\mathbf{q}'\times\mathbf{p}}{(E+M)} - \frac{\mathbf{p}'\cdot\mathbf{q} + i\boldsymbol{\sigma}\cdot\mathbf{p}'\times\mathbf{q}}{(E'+M)}]\Big\}\chi_B^{(S)} \ . \tag{4.40}$$

To be able to compare results for form factors obtained from experiments it is often convenient to use the Sachs form factors $G_E(q^2)$ and $G_M(q^2)$ [41] which in the Breit frame, with $q_0 = E' - E = 0$ and $q^2 = \mathbf{q}^2$, can be expressed as

$$G_E(\mathbf{q}^2) = F_1(\mathbf{q}^2) - \frac{\mathbf{q}^2}{4M^2}F_2(\mathbf{q}^2) \ ,$$

$$G_M(\mathbf{q}^2) = F_1(\mathbf{q}^2) + F_2(\mathbf{q}^2) \ . \tag{4.41}$$

When including relativistic corrections to order $(\frac{1}{M^2})$, the charge density operator matrix element (4.40) can now in the Breit frame be written as

$$\rho^{em,B} = \chi_B^{(S')\dagger}\{G_E(\mathbf{q}^2)(1-\frac{\mathbf{q}^2}{8M^2}) + (2G_M(\mathbf{q}^2) - G_E(\mathbf{q}^2))\frac{i\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{p}}{4M^2}\}\chi_B^{(S)} \ . \tag{4.42}$$

If the momentum transfer $\mathbf{q}$ is small terms of order $(\frac{1}{M^2})$ will, however, be small and in the non-relativistic limit the charge density operator matrix element is simply reduced to

$$\rho^{em,B} = \chi_B^{(S')\dagger} G_E(\mathbf{q}^2)\chi_B^{(S)} \ . \tag{4.43}$$

The Fourier transform of $G_E(q^2)$ in the Breit frame is the charge density $\rho_E(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d^3q G_E(\mathbf{q}^2)e^{-i\mathbf{q}\cdot\mathbf{r}}$ or, equivalently, $G_E(\mathbf{q}^2) = \int d^3r \rho_E(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}$. On the other hand, the (mean square) charge radius is defined as $< r^2 >= \int d^3r r^2 \rho_E(\mathbf{r})$ so by expanding $e^{i\mathbf{q}\cdot\mathbf{r}}$ in powers of $\mathbf{q}\cdot\mathbf{r}$,

$$G_E(\mathbf{q}^2) = \int d^3r\rho_E(\mathbf{r}) - \frac{1}{6}q^2\int d^3r r^2\rho_E(\mathbf{r}) + O(q^4) \ , \tag{4.44}$$

one finally gets

$$\begin{aligned} < r^2 > &= -6\frac{dG_E(\mathbf{q}^2)}{dq^2}|_{q^2=0} \\ &= -6\frac{dF_1(\mathbf{q}^2)}{dq^2}|_{q^2=0} + \frac{3F_2(0)}{2M^2} \ . \end{aligned} \tag{4.45}$$

In the last step of Eq. (4.45) the relation between the Sachs electric form factor and the Dirac and Pauli form factors (Eq. (4.41)) was used. In other words, if the slope of the Dirac form factor at zero momentum transfer can be calculated from some model for the baryon and if the anomalous magnetic moment κ $(= F_2(0))$ is known, the (mean square) charge radius can be determined.

By assuming that the Dirac part F_1 of the form factor G_E for a baryon comes from the sum over three constituent (Dirac) quarks this (intrinsic) part can then be calculated by first adding all of the quark contributions $\rho_q^{(i)} = < p_i'|\rho(0)|p_i >$, $i = 1 \ \ldots \ 3$, (Eq. (4.9)) and then calculate

$$\begin{aligned} F_1(\mathbf{r}^2) &=< \Psi_{B'}|\sum_{i=1}^{3}\int\frac{d^3k_i}{(2\pi)^3}e^{i\mathbf{k}_i\cdot\mathbf{r}_i}(2\pi)^3\delta(\mathbf{k}_i-\mathbf{q})\rho_q^{(i)}(\mathbf{k}_i,\mathbf{q})|\Psi_B > \\ &=< \Psi_{B'}|\sum_{i=1}^{3}e^{i\mathbf{q}\cdot\mathbf{r}_i}\rho_q^{(i)}(\mathbf{q})|\Psi_B > \ . \end{aligned} \tag{4.46}$$

Relativistic corrections to the (Dirac) charge density operator of a quark can be calculated by introducing the velocity operator of a quark $\mathbf{v} = \frac{\mathbf{p}'+\mathbf{p}}{2m}$ and defining $\mathbf{p} = m\mathbf{v}-\frac{\mathbf{q}}{2}$, $\mathbf{p}' = m\mathbf{v}+\frac{\mathbf{q}}{2}$, where $\mathbf{q}$ is the momentum transfer. To lowest order in $\frac{1}{m^2}$, the charge density operator matrix element is then (if the flavor $\times$ spin notation $\eta\chi$ ($\eta^\dagger\chi^\dagger$) of the single quark operator is suppressed)

$$< p'|\rho(0)|p >= (1 - \frac{q^2}{8m^2} + \frac{i\boldsymbol{\sigma}\cdot\mathbf{q}\times(m\mathbf{v})}{4m^2})Q. \qquad (4.47)$$

When calculating the charge radius for ground state baryons the term linear in $\mathbf{q}$ will not contribute and can be left out.

If the baryon wave function $\Psi_B = \psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\phi\chi$ is symmetric with respect to spatial and combined flavor-spin coordinates and $e^{i\mathbf{q}\cdot\mathbf{r}_i}$ is expanded in powers of q^2, Eq. (4.46) simplifies to

$$F_1(\mathbf{q}^2) = 3 < \psi|1 - \frac{q^2}{8m^2} - \frac{q^2}{6}r_1^2 + O(q^4)|\psi >< \phi\chi|Q^{(1)}|\phi\chi > , \qquad (4.48)$$

where the upper index on Q indicates that the charge operator, denoted as $Q^{(1)} = \frac{1}{2\sqrt{3}}\lambda_8^{(1)} + \frac{1}{2}\lambda_3^{(1)}$, is acting on quark 1. The result for $F_1(\mathbf{q}^2)$ can then be used in Eq. (4.45). In Paper II the spatial part of the wave function was approximated as a harmonic oscillator wave function with an effective oscillator frequency that gives a nucleon radius that is consistent with the radius of the model in Ref. [31] and the Dirac part of the charge radii for the proton and the neutron was calculated. The results ($< r^2 >_{IA+rel}$) are given in Table II of Paper II along with the contributions from the anomalous magnetic moments ($< r^2 >_{an}$). For the proton the combined contribution $< r^2 >_{IA+rel} + < r^2 >_{an}$ will be $\sim$ 40% smaller than the experimental (squared) proton charge radius. The neutron charge radius, on the other hand, will get zero contribution from the impulse approximation combined with relativistic corrections, while the anomalous part is only $\sim$ 9% larger than the empirical (squared) neutron charge radius.

5. Exchange currents

5.1. The continuity equation

In the impulse approximation for the (constituent) quarks that constitute the baryons external currents are assumed to be absorbed by individual quarks that do not interact, and the resulting process is described by one-body operators. When going beyond this approximation, i.e., assuming interactions between the constituent quarks, the use of two-body operators could be necessary, depending on the form for the quark-quark interaction.

The continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{J} + i[H, \hat{\rho}] = 0 \ , \tag{5.1}$$

connects the electromagnetic current and charge (density) operators with the (non-relativistic) Hamiltonian. For a system of three constituent quarks the current (density) operator can be defined as

$$\mathbf{J} = \sum_{i=1}^{3} \mathbf{J}_i^q + \sum_{i<j} \mathbf{J}_{ij}^{ex} \ , \tag{5.2}$$

where $\mathbf{J}_i^q$ is a one-body current operator and $\mathbf{J}_{ij}^{ex}$ is a two-body or exchange current operator. In the case of three-quark interactions three-quark exchange currents also have to be considered. If these are, however, neglected the Hamiltonian for the system is

$$H = \sum_{i=1}^{3} T_i + \sum_{i<j} V_{ij} \ . \tag{5.3}$$

Here T_i is the one-body kinetic energy and V_{ij} is a two-body potential describing the quark-quark interaction. The continuity equation now separates into different equations for the one-body and two-body current operators, i.e.,

$$\boldsymbol{\nabla} \cdot \mathbf{J}_i^q + i[T_i, \hat{\rho}_i] = 0 \ , \tag{5.4a}$$

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{ij}^{ex} + i[V_{ij}, \hat{\rho}_i + \hat{\rho}_j] = 0 \ . \tag{5.4b}$$

In the above equations it is assumed that the charge density operator has only a one-body component. If there exists a two-body exchange charge density operator Eq. (5.4b) will get a small addition. If now the potential V_{ij} has such a form that $[V_{ij}, \hat{\rho}_i + \hat{\rho}_j] \neq 0$ this will imply that $\mathbf{J}_{ij}^{ex} \neq 0$, i.e., there are two-body corrections to the current $\mathbf{J}$. This is the case if the potential e.g. has a flavor dependence of the form

$$V_{ij} \sim \boldsymbol{\lambda}_i^F \cdot \boldsymbol{\lambda}_j^F \ , \tag{5.5}$$

as in the chiral pseudoscalar interaction mentioned in Section 3.5.1, since the charge density operator $\hat{\rho}_i$ is defined (in units of e) by

$$< p'|\hat{\rho}_i|p >= \bar{u}(p_i')Q^{(i)}\gamma_4 u(p_i) \ , \tag{5.6}$$

with $Q^{(i)}$ denoting the charge operator Q defined in Eq. (4.5) acting on quark i, and, consequently, $[\boldsymbol{\lambda}_i^F \cdot \boldsymbol{\lambda}_j^F, Q^{(i)}] \neq 0$.

5.2. Electromagnetic exchange current operators

To construct the exchange current operators a Lorentz-invariant photon-quark-quark vertex can be decomposed into single quark and many-body parts. The many-body part is further split into a reducible and an irreducible part. If the reducible part is taken as a part of the impulse approximation, the remaining irreducible part then give rise to exchange current operators. The above decomposition is given in momentum space in Figure. 5.1.

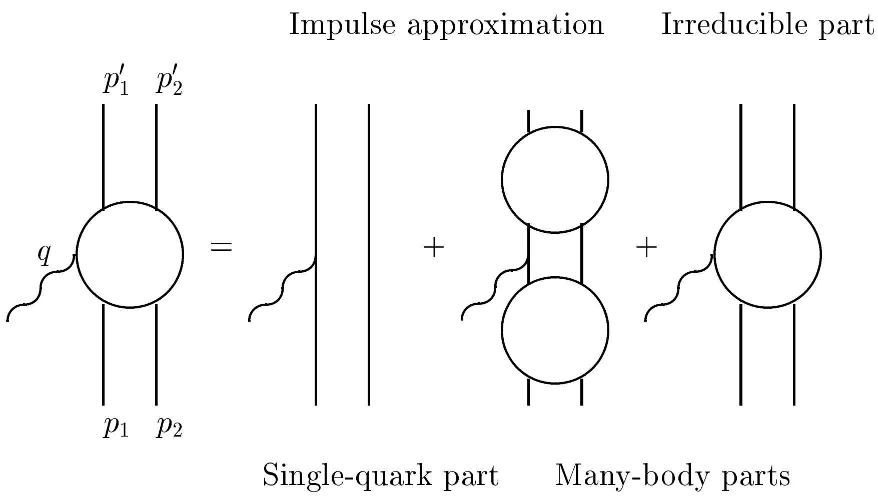
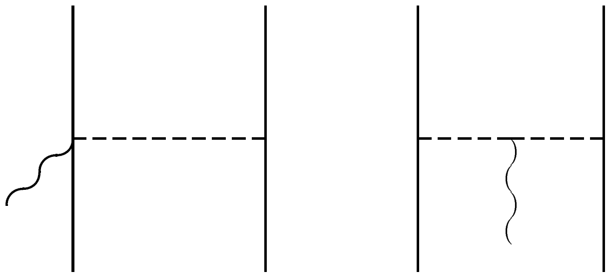


Figure 5.1: Decomposition of photon-quark-quark vertex.

The irreducible part of the diagram in Figure 5.1 can be decomposed into so called transition and diagonal terms, of which the first describes how the photon interacts with two different quarks and the second describes elastic photon-quark vertices. The transition type exchange currents can be calculated from the static limit of the Feynman diagram amplitudes. The so called Born terms of such terms are shown in Figure 5.2.



(a) (b)

Figure 5.2: Transition Born terms,
(a) contact Born term, (b) mesonic Born part.

Of special interest is the "external" exchange current or contact term in Figure 5.2(a), which can be calculated from the so called relativistic Born terms in Figure 5.3. The exchange current operator is constructed as a product of the single-particle current operator, the propagator $\tilde{p}$ for the intermediate particle in the diagrams of Figure 5.3(a) and the interaction V. In addition to the two diagrams in Figure 5.3(a) also the corresponding diagrams where quarks 1 and 2 are interchanged should be taken into account. The propagator $\tilde{p}$ can be written as

$$-\frac{1}{\gamma \cdot \tilde{p} - im} = -\frac{(\gamma \cdot \tilde{p} + im)}{(E - \tilde{p}_0)(E + \tilde{p}_0)}$$

$$= -\frac{(\boldsymbol{\gamma} \cdot \tilde{\mathbf{p}} + im + i\gamma_4 E)}{2E(E - \tilde{p}_0)} - \frac{(\boldsymbol{\gamma} \cdot \tilde{\mathbf{p}} + im - i\gamma_4 E)}{2E(E + \tilde{p}_0)} \ , \tag{5.7}$$

where, in the last step of Eq. (5.7), the propagator is divided into separate components corresponding to a particle with positive energy (quark) and a particle with negative energy (anti-quark) respectively. The positive energy part of the Born term is then subtracted, since this contribution is already included in the impulse approximation, and only the anti-quark component of the propagator for the intermediate quark is used in the calculation. The corresponding diagrams are shown in Figure 5.3(b). The exchange current operator is then model independent in the sense that it is related to the exchange potential by current conservation.

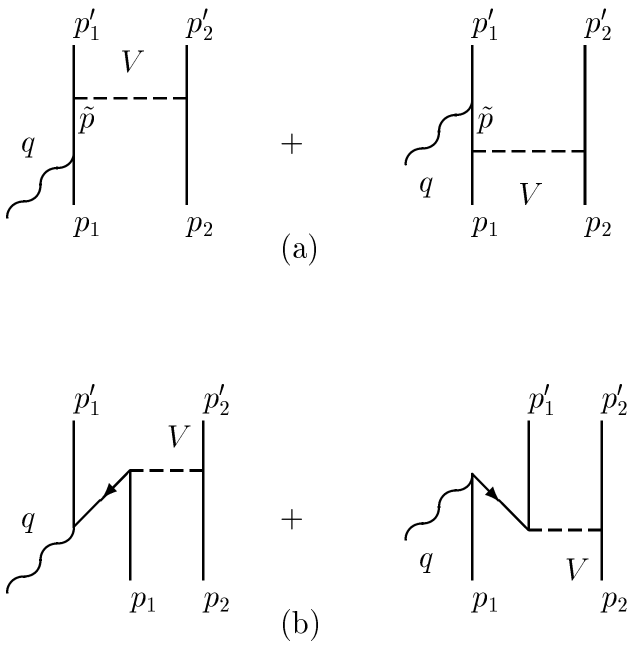


Figure 5.3: (a) Relativistic Born diagrams, (b) negative-energy components of the relativistic Born diagrams.

Another model independent exchange current operator is obtained from the "internal" radiation diagram in Figure 5.2(b). There could also be model dependent terms that are not shown in Figure 5.2.

If the quark-quark interaction V is energy independent and of such a form that it can be expressed in terms of five independent non-relativistic spin amplitudes that are the non-relativistic limits of five relativistic spin invariants, the exchange current operators obtained from the external and internal diagrams of Figure 5.2(a) and (b) will then satisfy the non-relativistic continuity equation with this interaction. In momentum space the continuity equation for the model independent exchange current to order $1/m^2$, where m is the constituent quark mass, can be written as

$$\begin{aligned} q_\mu J_\mu^{ex} &= V(\mathbf{p}_1', \mathbf{p}_2'; \mathbf{p}_1 + \mathbf{q}, \mathbf{p}_2)\rho_1(\mathbf{p}_1 + \mathbf{q}; \mathbf{p}_1) \\ &- \rho_1(\mathbf{p}_1'; \mathbf{p}_1' - \mathbf{q})V(\mathbf{p}_1' - \mathbf{q}, \mathbf{p}_2'; \mathbf{p}_1, \mathbf{p}_2) + [1 \leftrightarrow 2] \ , \end{aligned} \tag{5.8}$$

where $q_\mu = (\mathbf{q}, iq_0)$ is the photon four-momentum, the single-quark charge (density) operator is defined as in Eq. (5.6) and $[1 \leftrightarrow 2]$ denotes terms where the coordinates of quarks 1 and 2 have been interchanged. If the limit $q \rightarrow 0$ is taken the internal exchange current operator derived from the diagram in Figure 5.2(b) vanishes.

5.3. $SVTAP$-decomposition

If the quark-quark interaction is energy independent and depends only on the invariant momentum transfer $t = -(p_1' - p_1)^2$, its Lorentz-invariant version can be written as a linear combination of five relativistic spin invariants, the so called Fermi invariants $SVTAP$, where [75]

$$S = 1^{(1)}1^{(2)} \ , \quad V = \gamma_\mu^{(1)}\gamma_\mu^{(2)} \ , \quad T = \frac{1}{2}\sigma_{\mu\nu}^{(1)}\sigma_{\mu\nu}^{(2)} \ ,$$

$$A = i\gamma_5^{(1)}\gamma_\mu^{(1)}i\gamma_5^{(2)}\gamma_\mu^{(2)} \ , \quad P = \gamma_5^{(1)}\gamma_5^{(2)} \ , \tag{5.9}$$

so that

$$V(\mathbf{p}_1', \mathbf{p}_2'; \mathbf{p}_1, \mathbf{p}_2) = \bar{u}(p_1')\bar{u}(p_2')\{\sum_{j=1}^{5}[v_j^+(t) + v_j^-(t)\boldsymbol{\lambda}_1^F \cdot \boldsymbol{\lambda}_2^F]F_j\}u(p_1)u(p_2) \ . \tag{5.10}$$

Here F_j, $j = 1 \ldots 5$ are the relativistic Fermi invariants in the order $SVTAP$ while $v_j^+(t)$ and $v_j^-(t)$ are functions associated with the flavor independent and flavor dependent parts of the interaction respectively.

If now the non-relativistic quark-quark potential is expressed as

$$\tilde{V}(r) = \sum_{j=1}^{5}[\tilde{w}_j^+(r) + \tilde{w}_j^-(r)\boldsymbol{\lambda}_1^F \cdot \boldsymbol{\lambda}_2^F]\tilde{\Omega}_j \ , \tag{5.11}$$

with the spin operators $\tilde{\Omega}_j$ defined as

$$\tilde{\Omega}_1 = 1 \ , \quad \tilde{\Omega}_2 = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{L} \ , \quad \tilde{\Omega}_3 = 3 \ \boldsymbol{\sigma}_1 \cdot \hat{r} \ \boldsymbol{\sigma}_2 \cdot \hat{r} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \ ,$$

$$\tilde{\Omega}_4 = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \ , \quad \tilde{\Omega}_5 = \frac{1}{2}(\boldsymbol{\sigma}_1 \cdot \mathbf{L} \ \boldsymbol{\sigma}_2 \cdot \mathbf{L} + \boldsymbol{\sigma}_2 \cdot \mathbf{L} \ \boldsymbol{\sigma}_1 \cdot \mathbf{L}) \ , \tag{5.12}$$

the interaction in momentum space will be of the form

$$V(k) = \int d^3 r e^{-i\mathbf{k}\cdot\mathbf{r}}\tilde{V}(r) = \sum_{j=1}^{5}[w_j^+(k) + w_j^-(k)\boldsymbol{\lambda}_1^F \cdot \boldsymbol{\lambda}_2^F]\Omega_j \ . \tag{5.13}$$

The spin operators Ω_j will here be

$$\Omega_1 = 1 \ , \quad \Omega_2 = \frac{1}{2}i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{k} \times \mathbf{P} \ , \quad \Omega_3 = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \ k^2 - 3 \ \boldsymbol{\sigma}_1 \cdot \mathbf{k} \ \boldsymbol{\sigma}_2 \cdot \mathbf{k} \ ,$$

$$\Omega_4 = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \ , \quad \Omega_5 = \boldsymbol{\sigma}_1 \cdot \mathbf{k} \times \mathbf{P} \ \boldsymbol{\sigma}_2 \cdot \mathbf{k} \times \mathbf{P} \ , \tag{5.14}$$

with $\mathbf{k} = \mathbf{p}_1' - \mathbf{p}_1 = -(\mathbf{p}_2' - \mathbf{p}_2)$ and $\mathbf{P} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p'}_1) = -\frac{1}{2}(\mathbf{p}_2' + \mathbf{p}_2)$. If the functions $\tilde{w}_i^{\pm}(r)$ are known, the corresponding components in momentum space, $w_i^{\pm}(k)$, can be obtained. To get a relation between $w_i^{\pm}(k)$ and the potential coefficient $v_j^{\pm}(k)$ in the Fermi invariant decomposition the non-relativistic limits of the Fermi invariants F_j are calculated to order $1/m^2$ and expressed as linear combinations of the spin operators Ω_i in Eq. (5.14), i.e.,

$$F_j = \sum_{k=1}^{5} A_{jk}\Omega_k \ . \tag{5.15}$$

The Fermi-invariant potential components can then be written as

$$v_j^{\pm}(k) = \sum_{k=1}^{5} (A^{-1})^T_{jk} w_k^{\pm}(k) \ , \tag{5.16}$$

following the method developed by Blunden and Riska [76] for nucleon-nucleon interactions.

When the interaction is thus written as a linear superposition of Fermi invariants, the exchange current operators are constructed as a corresponding linear combination of exchange current operators for the Fermi invariants. In Ref. [77] the exchange current operators corresponding to the Fermi invariants have been given for a two-nucleon system. The corresponding operators for a system of two quarks can then be obtained by substituting the nucleon mass by the (constituent) quark mass and all isospin matrices $\tau^{(i)}$ by flavor matrices $\lambda^{(i)}$.

5.4. Exchange current contributions to the magnetic moment

5.4.1. The exchange magnetic moment operator

From the model independent electromagnetic exchange current (density) operators that correspond to the Fermi invariants it is possible to calculate exchange magnetic moment operators that will add to the one-body magnetic moment operator. The exchange current density operators are of the form $\mathbf{J}^{ex}(\mathbf{k}_1, \mathbf{k}_2; \mathbf{P}_1, \mathbf{P}_2)$, where $\mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i$ and $\mathbf{P}_i = \frac{1}{2}(\mathbf{p}_i' + \mathbf{p}_i)$. Now the exchange magnetic moment operators can be calculated as [77]

$$\boldsymbol{\mu}_{ex}(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2)} [\mathbf{M} + \frac{1}{2}\mathbf{R}_{12} \times \mathbf{N}] \ , \tag{5.17}$$

where $\mathbf{k} = \mathbf{k}_1 = -\mathbf{k}_2$, $\mathbf{R}_{12}$ is the center-of-mass coordinate for the quark pair, and

$$\mathbf{M}(\mathbf{k}; \mathbf{P}_1, \mathbf{P}_2) = -\frac{1}{2} i \lim_{\mathbf{q}\to 0} \boldsymbol{\nabla}_{\mathbf{q}} \times \mathbf{J}^{ex}(\frac{1}{2}\mathbf{q} + \mathbf{k}, \frac{1}{2}\mathbf{q} - \mathbf{k}; \mathbf{P}_1 + \mathbf{P}_2) \ , \tag{5.18a}$$

$$\mathbf{N}(\mathbf{k}; \mathbf{P}_1, \mathbf{P}_2) = \lim_{\mathbf{q}\to 0} \mathbf{J}^{ex}(\frac{1}{2}\mathbf{q} + \mathbf{k}, \frac{1}{2}\mathbf{q} - \mathbf{k}; \mathbf{P}_1, \mathbf{P}_2) \ . \tag{5.18b}$$

In Eq. (5.18) the notation $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2$ is used. For quarks that combine to S-state baryons the second term in Eq. (5.17) will give no contribution.

5.4.2. The static exchange magnetic moment

The exchange magnetic moment operators of a two-quark system can be obtained from the results derived for a two-nucleon system [77] by suitable substitutions as mentioned above. Thus if a term in the two-nucleon case has an isospin dependence of the form $(\boldsymbol{\tau}^{(i)} \times \boldsymbol{\tau}^{(j)})_3$ the flavor dependence of the corresponding term in the SU(3) case can be obtained by the substitution

$$\begin{aligned}(\boldsymbol{\tau}^{(i)} \times \boldsymbol{\tau}^{(j)})_3 &= [\tau_1^{(i)}\tau_2^{(j)} - \tau_2^{(i)}\tau_1^{(j)}] \\ &\longrightarrow [\lambda_1^{(i)}\lambda_2^{(j)} - \lambda_2^{(i)}\lambda_1^{(j)} + \lambda_4^{(i)}\lambda_5^{(j)} - \lambda_5^{(i)}\lambda_4^{(j)}] \ . \end{aligned} \tag{5.19}$$

If the hyperfine interaction between two quarks is now taken to be

$$V(\mathbf{r}_{ij}) = \frac{1}{3} f(r_{ij}) \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)} \boldsymbol{\lambda}^{(i)} \cdot \boldsymbol{\lambda}^{(j)} \ , \tag{5.20}$$

where the expression $\boldsymbol{\lambda}^{(i)} \cdot \boldsymbol{\lambda}^{(j)}$ above should be understood as $\sum_{k=1...8} \lambda_k^{(i)}\lambda_k^{(j)}$, the exchange magnetic moment operator for ground state baryons will to lowest order in v/c, assuming that the interaction is due to pseudoscalar exchange mechanisms, have the form

$$\boldsymbol{\mu}_{ex,ij} = -\frac{1}{2} g(r_{ij}) \{\lambda_1^{(i)}\lambda_2^{(j)} - \lambda_2^{(i)}\lambda_1^{(j)} + \lambda_4^{(i)}\lambda_5^{(j)} - \lambda_5^{(i)}\lambda_4^{(j)}\} \boldsymbol{\sigma}^{(i)} \times \boldsymbol{\sigma}^{(j)} \ , \tag{5.21}$$

where $g(r_{ij})$ can be calculated from $f(r_{ij})$ as

$$g(r) = -\frac{1}{3}\{2\int_r^\infty dr' r' f(r') - \frac{1}{r}\int_r^\infty dr' \int_{r'}^\infty dr'' r'' f(r'')\} \ . \tag{5.22}$$

If the flavor dependent part of the potential used is of the form described in Eq. (3.42), i.e., with different contributions $V_\gamma(r_{ij})$, $\gamma = \pi,\ K,\ \eta$, corresponding to pion, kaon and η exchange, this has to be taken into account in the exchange magnetic moment operator by combining the proper functions with the corresponding flavor dependent operators. The two different parametrizations (3.48) and (3.49) for $V_\gamma(r_{ij})$, corresponding to different versions of the chiral constituent quark model, give different results. If the parametrization (3.48) of Ref. [30] is used, one has to further take into account that the hyperfine interaction cannot in this case be completely caused by pseudoscalar exchange mechanisms, since this interaction, expressed in momentum space, would not vanish with momentum. This requirement is met in the case of the second parametrization. In the first case, however, the exchange magnetic moment part that is due to pseudoscalar mechanisms can be estimated by calculating the fraction of the potential function (3.48) that would give zero volume integral (corresponding to a vanishing interaction in the zero momentum limit). The remaining part of the potential function will then be caused by other exchange mechanisms. These, however, do not contribute to the exchange magnetic moment to lowest order in v/c [77]. The exchange magnetic moment contributions from the pseudoscalar exchange part were calculated for the ground state baryons and the Δ^{++} and Ω^- hyperons in the parametrization (3.48) and the results are given in the column EXCI of Table 4 in Paper I. For the second parametrization the corresponding values for the proton and the neutron have been calculated as 0.18 and -0.18 n.m., respectively [66].

5.4.3. Relativistic corrections to the exchange magnetic moment

The exchange magnetic moment operator calculated in Eq. (5.21) is based on the static non-relativistic pseudoscalar exchange interaction and will give no contribution for the decuplet baryons due to their flavor symmetry, nor for the Σ^- and Ξ^-. By taking into account the lowest order corrections from the relativistic form for the pseudoscalar exchange interaction further terms will appear, and their contribution can be written as

$$\mathbf{M}_{ex,rel} = \mathbf{M}_{ex}^{F,S} + \mathbf{M}_{ex}^{F,A} \ , \tag{5.23}$$

where $\mathbf{M}_{ex}^{F,S}$ is a flavor symmetric part and $\mathbf{M}_{ex}^{F,A}$ a flavor antisymmetric part. In the SU(2) case the isospin operator in the symmetric part of a two-nucleon system is of the form [77] $e^{(i)}\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)} + \boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}e^{(i)} = \{e^{(i)}, \boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\}$, where $e^{(i)} = (1+\tau_3^{(i)})/2$. The corresponding SU(3) operator will be

$$\begin{aligned}
&\{e^{(i)}, \boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\} \longrightarrow \{Q^{(i)}, \boldsymbol{\lambda}^{(i)}\cdot\boldsymbol{\lambda}^{(j)}\} \\
&= \frac{1}{2}(\frac{1}{\sqrt{3}}\lambda_8^{(i)} + \lambda_3^{(i)})\sum_{a=1}^{8}\lambda_a^{(i)}\lambda_a^{(j)} + \sum_{a=1}^{8}\lambda_a^{(i)}\lambda_a^{(j)}\frac{1}{2}(\frac{1}{\sqrt{3}}\lambda_8^{(i)} + \lambda_3^{(i)}) \\
&= \frac{2}{3}(\lambda_3^{(i)} + \frac{1}{\sqrt{3}}\lambda_8^{(i)}) + \frac{1}{\sqrt{3}}(\lambda_8^{(i)}\lambda_3^{(j)} + \lambda_3^{(i)}\lambda_8^{(j)}) + \frac{1}{3}\sum_{a=1}^{5}\lambda_a^{(i)}\lambda_a^{(j)} \\
&\qquad -\frac{2}{3}\sum_{a=6}^{7}\lambda_a^{(i)}\lambda_a^{(j)} - \frac{1}{3}\lambda_8^{(i)}\lambda_8^{(j)} \ .
\end{aligned} \tag{5.24}$$

To obtain the SU(3) version of the isospin operator for the antisymmetric part, which is of the form [77] $(\boldsymbol{\tau}^{(i)} - \boldsymbol{\tau}^{(j)})_3$, the substitution

$$(\boldsymbol{\tau}^{(i)} - \boldsymbol{\tau}^{(j)})_3 = [\tau_3^{(i)} - \tau_3^{(j)}]$$

$$\longrightarrow [\lambda_3^{(i)} - \lambda_3^{(j)} + \frac{1}{\sqrt{3}}(\lambda_8^{(i)} - \lambda_8^{(j)})] \tag{5.25}$$

should be made. The first term of Eq. (5.23) now gives a contribution also to the exchange magnetic moment of the decuplet baryons. The results for the contribution (5.23), calculated in the parametrization (3.48) when taking into account only the fraction coming from pseudoscalar exchange mechanisms, are given for the ground state baryons and the Δ^{++} and Ω^- hyperons in column EXCII of Table 4 in Paper I. The corresponding values for the proton and the neutron are 0.16 and -0.09 n.m. respectively [66] in the second parametrization.

A substantial part of the potential in the model of Eq. (3.48) has to be interpreted as coming from some short range exchange mechanism. One possibility is to ascribe this part to axial vector exchange mechanisms (similar results are also attained if tensor exchange mechanisms are assumed). A contribution from this part, having the same order in v/c as (5.23), will have a flavor dependence that is similar to (5.23) but with an additional term that in SU(2) has an isospin operator of the form $e^{(i)}\boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)} - \boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)} e^{(i)} = [e^{(i)}, \boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)}]$ [77]. The corresponding SU(3) operator can be obtained by the substitution

$$\begin{aligned} &[e^{(i)}, \boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)}] \longrightarrow [Q^{(i)}, \boldsymbol{\lambda}^{(i)} \cdot \boldsymbol{\lambda}^{(j)}] \\ &= -i(\lambda_1^{(i)}\lambda_2^{(j)} - \lambda_2^{(i)}\lambda_1^{(j)} + \lambda_4^{(i)}\lambda_5^{(j)} - \lambda_5^{(i)}\lambda_4^{(j)}) \ . \end{aligned} \tag{5.26}$$

The contribution to the exchange magnetic moment from this phenomenological short-range interaction in the parametrization (3.48) is given in column EXCIII of Table 4 in Paper I.

5.4.4. Confinement contributions to the magnetic moment

Also the confining interaction $v_C(r)$ will give a contribution to the magnetic moment of the baryons. This interaction may formally be viewed as the static approximation to a relativistic scalar exchange interaction with positive sign. For the model of Ref. [30]

$$v_C(r) = Cr \ , \tag{5.27}$$

with $C = 0.474$ fm^{-2}, and the corresponding contributions for the ground state baryons and the Δ^{++} and Ω^- hyperons are given in column CONF of Table 4 in Paper I. Corresponding values for the proton and the neutron, when using the model of Ref. [31] (with a appropriate harmonic wave function approximation), where

$$v_C(r) = C'r + V_0 \ , \tag{5.28}$$

with $C' = 2.33$ fm^{-2} and $V_0 = -416$ MeV, are 0.88 and -0.59 n.m. respectively [66].

5.4.5. The total magnetic moment

By adding all the exchange magnetic moment contributions to the result from the impulse approximation with relativistic corrections discussed in Chapter 4 one finally yields a value for the total magnetic moment of the baryons. The unfortunate reduction of the impulse approximation result, due to the relativistic corrections in the one-body operator, is now compensated for and the final result is in good agreement with data. The results for the model of Ref. [30], with the parametrizations (3.48) for the hyperfine interaction and (5.27) for the confining interaction, are given in column Total of Table 4 in Paper I. The total magnetic moments for the proton and the neutron in the model of Ref. [31], with parametrizations (3.49) and (5.28), are related in the discussion part of Paper II.

5.5. Exchange current contributions to the axial current

As in the electromagnetic exchange current case it is also possible to construct axial vector exchange current operators for constituent quarks. The "model-independent" part of this axial exchange current is constructed from the quark-quark interaction. This calculation can be compared to the corresponding derivation in the nucleon-nucleon case [78]. One takes as the starting point the relativistic Born terms (Figure 5.3(a)), where the axial field couples to one of the quarks in a quark-quark system. The axial exchange current operator is then the product of the single-quark operator, the propagator of the intermediate quark (Eq. (5.7)) and the interaction V. The positive-energy part of the propagator will be discarded to avoid double counting. By taking the non-relativistic reduction of the operator one then yields the contact term of Figure 5.2(a). For the interaction V one uses the $SVTAP$-decomposition of Section 5.3, with one modification concerning the P-invariant. This invariant should be replaced by a "pseudovector" invariant $P' = \frac{1}{4m^2}(\gamma_\mu^{(1)} \cdot k_\mu)\gamma_5^{(1)}(\gamma_\mu^{(2)} \cdot k_\mu)\gamma_5^{(2)}$ to avoid double counting of a contribution associated with a possible $\rho\pi$-exchange component. The axial exchange current operator will now have contributions from the invariants $SVTA$ of order $\left(\frac{1}{m^3}\right)$, which may give corrections to the predicted value of $g_A(0)$ for the baryons if the model used for quark-quark interaction includes exchange mechanisms of the above types, while the P'-invariant will give a contribution of order $\left(\frac{1}{m^5}\right)$ and thus can be neglected. "Model-dependent" parts of the axial exchange current operator may also give corrections to $g_A(0)$.

5.6. Exchange current contributions to the charge radius

5.6.1. The exchange charge density operator

The exchange charge density operator corresponding to the sum of the contact term in Figure 5.2(a) and a similar diagram with quarks 1 and 2 interchanged can be constructed in the following way. The two contact terms will get contributions from the non-relativistic limit of four external coupling diagrams, two of which are shown in Figure 5.3(b). The general form of the exchange charge density operator is obtained by multiplying the single-quark charge density operator with the propagator of the intermediate quark and the interaction V. As in Section 5.2 only the negative-energy part of the propagator for the intermediate quark is taken into account to avoid double counting. The exchange charge density operators corresponding to the $SVTAP$-decomposition of the quark-quark potential are then constructed.

The exchange charge density operators will to lowest order in (v/c) have the general form

$$\begin{aligned} \rho_j(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}) &= \rho_j(\mathbf{k}_2, \mathbf{q}) + \rho_j(\mathbf{k}_1, \mathbf{q}) \\ = \frac{1}{8m^3} \Big\{ O_j^+ \Big(& 2Q^{(1)} v_j^+(\mathbf{k}_2) \\ & + \{Q^{(1)}, \sum_{k=1}^{8} \lambda_k^{(1)} \lambda_k^{(2)}\} v_j^-(\mathbf{k}_2) \Big) \\ & - O_j^- [Q^{(1)}, \sum_{k=1}^{8} \lambda_k^{(1)} \lambda_k^{(2)}] v_j^-(\mathbf{k}_2) \Big\} \\ & + (1 \leftrightarrow 2) , \quad j = 1, \ldots, 5 . \end{aligned} \tag{5.29}$$

Here $O_j^\pm$ are operators related to the $SVTAP$-decomposition, $Q^{(i)}$ is the (SU(3)) electric charge operator acting on quark i, and v_j^+ and v_j^- are flavor independent and flavor dependent potentials, respectively. The operators O_j^+ are defined as

$$\begin{aligned} O_1^+ &= \mathbf{q^2} + 2i\boldsymbol{\sigma}^{(1)} \cdot \mathbf{P_1} \times \mathbf{q} , \\ O_2^+ &= \mathbf{q} \cdot \mathbf{k}_2 + [\boldsymbol{\sigma}^{(2)} \times \mathbf{k}_2] \cdot [\boldsymbol{\sigma}^{(1)} \times \mathbf{q}] + 2i\boldsymbol{\sigma}^{(1)} \cdot \mathbf{P_2} \times \mathbf{q} , \\ O_3^+ &= \mathbf{q} \cdot \mathbf{k}_2 + \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} \mathbf{q} \cdot \mathbf{k}_2 - \boldsymbol{\sigma}^{(2)} \cdot \mathbf{q} \boldsymbol{\sigma}^{(1)} \cdot (\mathbf{k}_2 - \mathbf{q}) + 2i\boldsymbol{\sigma}^{(2)} \cdot \mathbf{P_2} \times \mathbf{q} , \\ O_4^+ &= \mathbf{q^2} \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} - \boldsymbol{\sigma}^{(1)} \cdot \mathbf{q} \boldsymbol{\sigma}^{(2)} \cdot \mathbf{q} + \boldsymbol{\sigma}^{(1)} \cdot \mathbf{q} \boldsymbol{\sigma}^{(2)} \cdot \mathbf{k}_2 + 2i\boldsymbol{\sigma}^{(2)} \cdot \mathbf{P_2} \times \mathbf{q} , \\ O_5^+ &= \boldsymbol{\sigma}^{(1)} \cdot \mathbf{q} \boldsymbol{\sigma}^{(2)} \cdot \mathbf{k}_2 , \end{aligned} \tag{5.30}$$

and the corresponding operators O_j^- as

$$
\begin{aligned}
O_1^- &= 2\mathbf{q}\cdot\mathbf{P_1}\ , \\
O_2^- &= 2\mathbf{q}\cdot\mathbf{P_2} - i[\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}]\cdot\mathbf{q}\times\mathbf{k}_2\ , \\
O_3^- &= 2\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{P_1} + 2[\boldsymbol{\sigma}^{(1)}\times\mathbf{q}]\cdot[\boldsymbol{\sigma}^{(2)}\times\mathbf{P_2}] - i[\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}]\cdot\mathbf{q}\times\mathbf{k}_2\ , \\
O_4^- &= 2[\boldsymbol{\sigma}^{(1)}\times\mathbf{q}]\cdot[\boldsymbol{\sigma}^{(2)}\times\mathbf{P_1}]\ , \\
O_5^- &= 0\ .
\end{aligned}
\tag{5.31}
$$

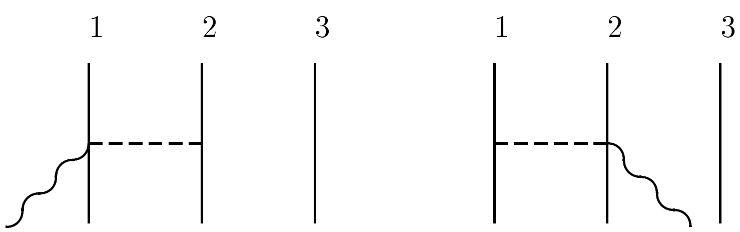
By adding the operators ρ_j, $j=1,\ldots,5$, the total exchange charge density operator for the contact terms is obtained.

Another possible contribution to the exchange charge density operators can be obtained from the diagram in Figure 5.2(b), the so called mesonic part of the Born diagram. From this diagram an exchange charge density operator corresponding to the flavor dependent part of the interaction can be calculated. This operator will be proportional to the energy transfer of the exchanged meson [77, 79] and if the energy transfer is assumed to be vanishingly small in the interaction model there will be no contribution to the exchange charge density from this diagram.

5.6.2. Exchange current contributions to the baryon charge radius

Since the exchange charge density operator (5.29) is of order m^{-3}, or, equivalently, of order $(\frac{v}{c})^3$ while the interaction in the $SVTAP$-decomposition is of order $(\frac{v}{c})^2$, the continuity equation that links the exchange charge density operators of a given order in $(\frac{v}{c})$ to terms of the next order in $(\frac{v}{c})$ in the interaction will not put any restraints on the exchange charge density operator. There will however be another constraint, namely that the total charge must be conserved. Thus the contribution from the exchange charge density must have a vanishing volume integral.

From the contact terms in a three-quark system six terms will contribute to the exchange charge density operator.



(a)

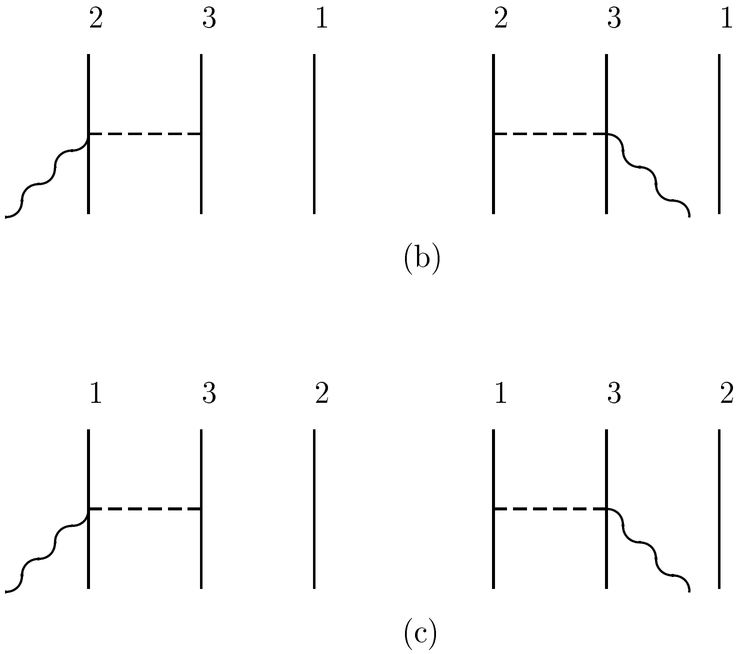


Figure 5.4: Contact terms in a three-quark system with exchanges between (a) quarks 1 and 2, (b) quarks 2 and 3, (c) quarks 1 and 3.

The six terms can be arranged two by two, as shown in Figure 5.4, corresponding to the exchange charge density operators for three two-quark systems. Hence, the total exchange charge density operator can be written as

$$\rho_{ex} = \rho_{ex}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}_{12}) + \rho_{ex}(\mathbf{k}_2, \mathbf{k}_3, \mathbf{q}_{23}) + \rho_{ex}(\mathbf{k}_1, \mathbf{k}_3, \mathbf{q}_{13}) \ , \tag{5.32}$$

where $\mathbf{q}_{ij}$ is the momentum of the (virtual) photon incident on the system of quarks i and j and where $\rho_{ex}(\mathbf{k}_i, \mathbf{k}_j, \mathbf{q}_{ij})$ can be calculated as in the previous section.

If the combined spatial, spin and flavor part of the three-quark system is symmetric with respect to all three quarks, the total contribution to the charge radius of the system is just three times the contribution from one of the terms in Eq. (5.32), e.g. $\rho_{ex}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}_{12}) = \rho_{ex}(\mathbf{k}_2, \mathbf{q}) + (1 \leftrightarrow 2)$. To calculate the baryon (mean square) charge radius one therefore defines an exchange charge form factor

$$F_{C,ex}(q^2) = 3 < e^{i\mathbf{q}\cdot\mathbf{r}_1} \int \frac{d^2k_2}{(2\pi)^3} e^{-i\mathbf{k}_2\cdot\mathbf{r}_{12}} \rho_{ex}(\mathbf{k}_2, \mathbf{q}) > +(1 \leftrightarrow 2) \ , \tag{5.33}$$

where $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$. The factor 3 in front of the matrix element comes from the wave function symmetry. The contribution to the charge radius will then be

$$< r^2 >_{ex} = -6 \frac{dF_{C,ex}(q^2)}{d(q^2)}|_{q^2=0} \;, \tag{5.34}$$

and this contribution should be added to the result of the impulse approximation with relativistic corrections.

In the calculations of $< r^2 >_{ex}$ for ground-state baryons the exchange charge density operators (Eq. (5.29)) can be considerably simplified. All terms involving $\mathbf{P}_i = \frac{1}{2}(\mathbf{p}_i + \mathbf{p}_i') = m\mathbf{v}_i$, $i = 1,\ 2$, where $\mathbf{p}_i(\mathbf{p}_i')$ is the initial(final) momentum of quark i will be small. Furthermore, terms of the form $\boldsymbol{\sigma}^{(1)} \cdot \mathbf{a}\boldsymbol{\sigma}^{(2)} \cdot \mathbf{b}$ can be written as a sum of a scalar and a tensor part, $\boldsymbol{\sigma}^{(1)} \cdot \mathbf{a}\boldsymbol{\sigma}^{(2)} \cdot \mathbf{b} = \frac{1}{3}\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}\mathbf{a} \cdot \mathbf{b} - \frac{1}{3}(\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)}\mathbf{a} \cdot \mathbf{b} - 3\boldsymbol{\sigma}^{(1)} \cdot \mathbf{a}\boldsymbol{\sigma}^{(2)} \cdot \mathbf{b})$ and for ground-state baryons the tensor part will not contribute to the exchange charge form factor. With these simplifications terms involving O_j^- defined in Eq. (5.31) will be reduced so that only O_2^- and O_3^- are nonzero, both being of the form $-i(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}) \cdot \mathbf{q} \times \mathbf{k}_2$, which should be combined with the flavor dependent factor $[Q^{(1)}, \sum_{k=1}^{8} \lambda_k^{(1)} \lambda_k^{(2)}]$ given in Eq. (5.26). It can be shown that when this spin-flavor operator is applied to the combined spin-flavor wave functions of the ground-state baryons it will give zero contribution to the exchange charge form factor. Thus for ground-state baryons only terms involving the operators O_j^+ in Eq. (5.29) will eventually contribute to the charge radius. These are therefore the only ones included in Paper II, where the exchange charge radii of the proton and the neutron were calculated.

In Paper II the chiral constituent quark model of Ref. [31] was used in combination with a harmonic oscillator wave function approximation of the original wave function in Ref. [31]. The exchange current contributions to the charge radius in this model come from the flavor independent linear confining interaction (5.28) and the flavor dependent hyperfine interaction (3.42) with the parametrization (3.49). The only potential functions $v_j^{\pm}$ that are nonzero in Eq. (5.29) for the exchange charge operator in this model are therefore v_1^+ and v_5^-. The potential v_1^+ is related to the confining interaction, which is interpreted as a static approximation to a scalar (S) exchange interaction [80] (with opposite sign compared to the conventional scalar exchange interaction), and it can be obtained as the inverse Fourier transform of the confining interaction v_c in Eq. (5.28). The potential $v_5^-(\mathbf{k})$, on the other hand, is related to the hyperfine interaction which in this model is assumed to have its origin solely in pseudoscalar (P) exchange mechanisms. It can be calculated as the inverse Fourier transform of a potential $\tilde{v}_\gamma(r)$, which in Paper I was shown to be related to the potential $V_\gamma(r)$ of the hyperfine interaction by $V_\gamma(r) = \frac{1}{3}\boldsymbol{\nabla}^2\tilde{v}_\gamma(r)$. The results for the contributions $< r^2 >_{CONF}$ and $< r^2 >_P$ to the proton and neutron charge radii are given in Table II of Paper II. Due to the operator structure of the exchange charge density operators the confinement contribution will be zero for the neutron but nonzero and positive for the proton. The contributions from pseudoscalar exchange mechanisms will be negative for both nucleons.

5.6.3. The total charge radius

When the impulse approximation results (including relativistic corrections) for the Dirac part of the squared charge radius are combined with the exchange current contributions and the anomalous Pauli part (cf. Eq. (4.45)) the mean-square charge radii of the proton and the neutron are 0.61 fm^2 and -0.136 fm^2, respectively. Comparison with the empirical values shows that the calculated value for the proton is $\sim$ 18% too small and, correspondingly, $\sim$ 16% too large for the neutron. It has been suggested [81, 82] that the mechanism which leads to the quarks acquiring their constituent mass would also cause the constituent quarks not to be point-like but to be quasiparticles that are spatially extended. One may thus introduce form factors for the constituent quarks by defining the quark charge operator acting on quark i as

$$Q^{(i)} = \frac{1}{2} F_3(q^2) \lambda_3^{(i)} + \frac{1}{2\sqrt{3}} F_8(q^2) \lambda_8^{(i)} \ , \tag{5.35}$$

where the form factors F_3 and F_8 should be equal to 1 for zero momentum transfer. These form factors will then give contributions to the total charge form factors of the nucleons and thus to the nucleon charge radii. The contribution can be expressed as linear combinations of the (squared) up and down quark radii, $< r^2 >_u$ and $< r^2 >_d$, respectively.

A natural assumption would be to assume that the charge radii of the up and down quark are equal, which results in an addition $< r^2 >_q$ to the proton mean-square charge radius, the value of which can be adjusted so that the total charge radius of the proton ($< r^2 >_{IA+rel} + < r^2 >_{ex} + < r^2 >_{an} + < r^2 >_q$) is consistent with the empirical value, i.e. $< r^2 >_{tot,p} = < r^2 >_{exp,p} = 0.743$ fm^2. The up and down quark radius would then in the model of Ref. [31] be 0.36 fm. This, however, implies that the neutron will get no quark charge radius contribution, resulting as above in $< r^2 >_{tot,n} = -0.136$ fm^2, a value which still differs from the empirical one by 16 %. If, on the other hand, one allows $< r^2 >_u \neq < r^2 >_d$ it is possible to adjust the quark contribution from the up and down quarks to both the proton and the neutron charge radii, yielding $< r^2 >_{tot,p} = < r^2 >_{exp,p}$ and $< r^2 >_{tot,n} = < r^2 >_{exp,n}$. The radii for the up and down quarks are then 0.35 fm and 0.31 fm, respectively, in the parametrization of Ref. [31].

6. The irreducible π-gluon exchange interaction

6.1. Interactions between constituent quarks

As was suggested in Chapter 3 a hyperfine chiral pseudoscalar interaction associated with the spontaneously broken approximate chiral symmetry of QCD combined with a confining interaction can be used to explain some of the main features of the spectra of the nucleons and the hyperons, as e.g. the ordering of the low lying positive and negative parity states. The right ordering of these states is not achieved with the confining interaction combined with a color-magnetic hyperfine one-gluon exchange interaction alone. Indications that come from cooled lattice calculations [42] and calculations in the valence-QCD approximation [43, 44] suggest that the hyperfine one-gluon exchange interaction should be weak at low momentum transfer. In phenomenological studies [83, 84] of the color fine structure constant which describes the quark-gluon coupling strength it has been suggested that it freezes at small momentum transfer, i.e., reaches a constant (small) value, which would imply that the gluon acquires an effective, dynamical mass at low energies [85, 86]. A momentum dependent screening can therefore be introduced for the quark-gluon coupling constant, i.e. $\alpha_S(k^2) = \alpha_0 \cdot g(k^2)$, where $g(k^2)$ at high momenta is dominated by an inverse logarithmic fall-off factor that depends on the parameter Λ ($\sim$ the inverse screening length), and which will either vanish or reach a constant value below the QCD scale Λ_{QCD}. The parameters α_0 and Λ can be adjusted so that the effective coupling at the charmonium and bottomonium scales agrees with lattice calculation results [87]. Some parametrizations for $\alpha_S(k^2)$ are shown in Figure 6.1.

On the other hand, for the pseudoscalar interaction one should also introduce a factor that describes the decoupling of the pseudoscalar (approximate) Goldstone bosons from the constituent quarks for momenta above the chiral restoration scale Λ_χ. This can be done e.g. by multiplying the interaction (or the pion propagator) with a high-momentum cut-off function of monopole form, $f(k^2) = (\Lambda_\chi^2 - m_\pi^2)/(\Lambda_\chi^2 + k^2)$, which reaches unity for small momenta and zero for large momenta. The assumption of a weak nonzero gluon exchange interaction and a dominant pseudoscalar interaction will, however, imply the existence of an irreducible pseudoscalar-gluon interaction. For non-strange baryons one would then have irreducible π-gluon and η-gluon interactions, which will have components with the same flavor and spin dependence as the chiral pseudoscalar interaction.

6.2. The tensor component of the pseudoscalar interaction

The chiral pseudoscalar interaction (3.41) has a tensor component that does not contribute to the ground state baryon spectrum but which should

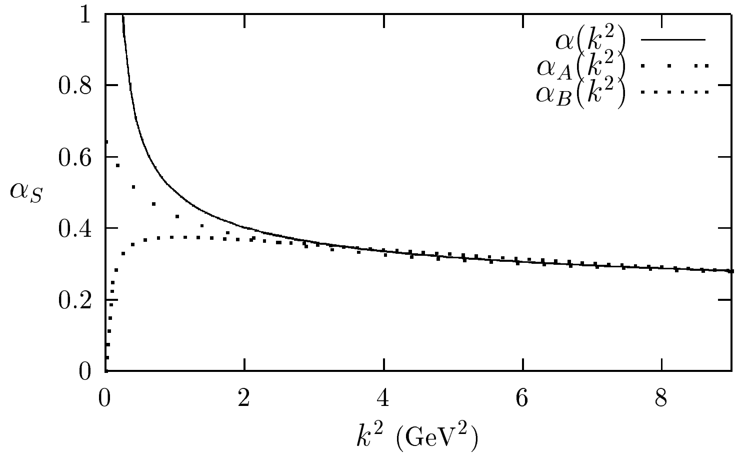


Figure 6.1: Parametrizations for the momentum dependence of the color fine structure constant α_s. Here $\alpha(k^2) = \frac{12\pi}{27}/\ln(\frac{k^2}{\Lambda^2})$, $\alpha_A(k^2) = \frac{12\pi}{27}/\ln(\frac{k^2+4m_g^2}{\Lambda^2})$ and $\alpha_B(k^2) = \alpha_0 \frac{k^2}{k^2+\Lambda^2}/[(1+\frac{k^2}{\Lambda_F^2})]$, with $\Lambda = 0.25$ GeV, $m_g = 0.37$ GeV, $\alpha_0 = 0.42$, $\Lambda_F = 4.3$ GeV.

be taken into account for higher states. By using the Yukawa form for this interaction it can be written as [29]

$$H_T = \sum_{i<j} \left\{ \sum_{a=1}^{3} V_T^{\pi}(r_{ij})\lambda_a^{(i)}\lambda_a^{(j)} + \sum_{a=4}^{7} V_T^{K}(r_{ij})\lambda_a^{(i)}\lambda_a^{(j)} + V_T^{\eta}(r_{ij})\lambda_8^{(i)}\lambda_8^{(j)} \right\} \hat{S}_{ij} \ , \tag{6.1}$$

where V_T^π, V_T^K and V_T^η describe the tensor potentials associated with π, K and η meson exchange, respectively, and having the form

$$V_T(r_{ij}) = \frac{\mathrm{g}^2}{4\pi} \cdot \frac{\mu^3}{12m_i m_j} \left(1 + \frac{3}{\mu r_{ij}} + \frac{3}{\mu^2 r_{ij}^2}\right) \frac{e^{-\mu r_{ij}}}{\mu r_{ij}} \ , \tag{6.2}$$

where g is a meson-quark coupling constant, μ is the mass of the exchanged meson and m is the constituent quark mass. In Eq. (6.1) $\hat{S}_{ij}$ is a shorthand notion for

$$\hat{S}_{ij} = 3\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ , \tag{6.3}$$

where $\hat{\mathbf{r}}_{ij}$ is a unit vector in the direction of the vector $\mathbf{r}_{ij}$.

For non-strange baryons only π and η mesons will give contributions to the tensor interaction. The η meson part will further be suppressed compared to the π meson part and subsequently the π meson mass, m_π, can be used in Eq. (6.2) for N and Δ states. For these states the quark-pion coupling constant g can be related to the nucleon-pion coupling constant $\mathrm{g}_{\pi N}$ as [29] $\mathrm{g} = \frac{3}{5}\frac{m_u}{m_N}\mathrm{g}_{\pi N}$, with $\frac{\mathrm{g}_{\pi N}^2}{4\pi} = 14.2$, thus giving $\frac{\mathrm{g}^2}{4\pi} = 0.67$. The effects of the tensor interaction will be small, giving rise to a small spin-orbit splitting e.g. in the $N = L = 1$ band. The empirical spin-orbit splittings in the baryon spectra are, however, very small or consistent with zero. One way of resolving this problem would be an inclusion of some other interaction that has a tensor component that counteracts the interaction (6.1). This will be the case for the irreducible π-gluon interaction.

6.3. The π-gluon exchange potential

When studying relativistic bound states of light particles the Bethe-Salpeter equation [46] may be taken as a starting point. The field-theoretical Lorentz-invariant elastic scattering amplitude M for two fermions in the center-of-momentum system is related to an interaction kernel K of irreducible diagrams and a two-particle (free) Green's function G as

$$M(p', p|W) = K(p', p|W) + \int \frac{d^4k}{(2\pi)^4} K(p', k|W) G(k|W) M(k, p|W) \ , \quad (6.4)$$

where p (p') is the four-momentum of one incoming (outgoing) fermion, $k = p' - p$ and W is the energy of one of the fermions in the c.m. frame.

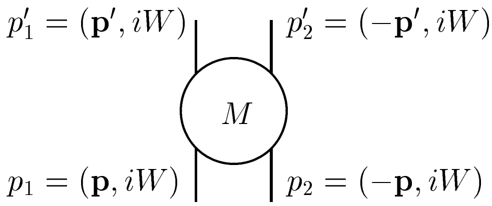


Figure 6.2: Fermion-fermion scattering in the center-of-momentum frame.

The relation between the field-theoretical scattering amplitude M and the so called S-matrix is given by

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2) \sqrt{\frac{m^4}{(2\pi)^{12} E_{p_1} E_{p_2} E_{p'_1} E_{p'_2}}} M_{fi} \ , \quad (6.5)$$

where the isospin dependence has been suppressed. Above the fermion mass is denoted by m, $E_p = \sqrt{\mathbf{p}^2 + m^2}$ and

$$M_{fi} = \bar{u}(\mathbf{p}_1')\bar{u}(\mathbf{p}_2')\ Mu(\mathbf{p}_1)u(\mathbf{p}_2)\ . \tag{6.6}$$

The Bethe-Salpeter (B-S) equation can formally be written as

$$M = K + KGM\ . \tag{6.7}$$

One would like to obtain a potential description of the two-fermion system, i.e. extract a potential from the field-theoretical description in Eq. (6.4). This can be done by first eliminating the time variable by making a three-dimensional reduction of the B-S equation. The Green's function, i.e. the two-fermion propagator, is written as a sum of a non-relativistic quasi-3-dimensional propagator $\tilde{g}$ and a residual part, $G - \tilde{g}$. If $\tilde{g}$ is chosen [45, 88] as a combination of a δ-function associated with the time variable and positive projection operators that reduce the B-S equation to a form that contains only positive-energy components one obtains for the field-theoretical amplitude

$$\begin{aligned} M(\mathbf{p}', \mathbf{p}|W) &= U(\mathbf{p}', \mathbf{p}|W) \\ &+ \int \frac{d^3k}{(2\pi)^3}\frac{m^2}{E_k}U(\mathbf{p}', \mathbf{k}|W)\frac{\Lambda_+^{(1)}(\mathbf{k})\Lambda_+^{(2)}(-\mathbf{k})}{E_k^2 - W^2 - i\varepsilon}M(\mathbf{k}, \mathbf{p}|W)\ , \end{aligned} \tag{6.8}$$

formally written as

$$M = U + U\tilde{g}M\ , \tag{6.9}$$

where U is a quasi-potential defined by

$$U = K + K(G - \tilde{g})U\ , \tag{6.10}$$

m is the fermion mass, $\mathbf{k} = \mathbf{p}' - \mathbf{p}$, $E_k = \sqrt{\mathbf{k}^2 + m^2}$ and $\Lambda_+^{(i)}$ is the positive energy projection operator, defined as

$$\Lambda_+^{(i)} = \frac{\gamma_4^{(i)}E_k - i\boldsymbol{\gamma}^{(i)}\cdot\mathbf{k} + m}{2m}\ . \tag{6.11}$$

The interaction kernel K can be approximated by

$$K = K^{(2)} + K^{(4)} + \ \dots\ , \tag{6.12}$$

where the first term is of second order in the coupling constant, the second term of fourth order etc., giving for the quasi-potential

$$U = K^{(2)} + K^{(4)} + K^{(2)}GK^{(2)} - K^{(2)}\tilde{g}K^{(2)} + \ \dots\ . \tag{6.13}$$

Now assume that the hyperfine interaction between two light constituent quarks consists of a dominant flavor dependent pseudoscalar interaction with spin-spin and tensor components of the form $\{V^\pi(r_{ij})\boldsymbol{\sigma}^{(i)}\cdot\ \boldsymbol{\sigma}^{(j)} + V_T^\pi(r_{ij})\hat{S}_{ij}\}$

$\boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)}$ (the η-component which gives a much smaller contribution is here suppressed) and a weak gluon interaction. Taking into account the π and gluon contributions the second order term will be $K^{(2)} = K_\pi^{(2)} + K_G^{(2)}$, and, subsequently, $K^{(4)} = K_{\pi\pi}^{(4)} + K_{GG}^{(4)} + K_{\pi G}^{(4)}$, where $K_{\pi G}^{(4)}$ is the irreducible π-gluon exchange contribution. The result for the potential is then to fourth order in the coupling constant

$$U \approx U_\pi + U_G + U_{\pi\pi} + U_{GG} + U_{\pi G} \ , \tag{6.14}$$

where the last term can be written as

$$\begin{aligned} U_{\pi G} &= K_{\pi G}^{(4)} + K_\pi^{(2)} G K_G^{(2)} + K_G^{(2)} G K_\pi^{(2)} - [K_\pi^{(2)} \tilde{g} K_G^{(2)} + K_G^{(2)} \tilde{g} K_\pi^{(2)}] \\ &= M_{\pi G}^{(4)} - [K_\pi^{(2)} \tilde{g} K_G^{(2)} + K_G^{(2)} \tilde{g} K_\pi^{(2)}] \ . \end{aligned} \tag{6.15}$$

The pseudo-potential for the irreducible π-gluon interaction can thus be calculated as the covariant field-theoretical amplitude for the combined π-gluon exchange from which has to be subtracted the iterated π and gluon exchange interaction. The potential $U_{\pi G}$ has the same flavor dependence as U_π and consists of central, spin-orbit, tensor and spin-spin components. These have been calculated in Paper III. The interaction $U_{\pi\pi}$ has been calculated in Ref. [89] and shown to enhance the effect of the isospin dependent spin-spin component of the one-pion exchange interaction while it seems to cancel the corresponding one-pion exchange tensor component. The interaction U_{GG}, on the other hand, is flavor independent and can be incorporated in the confining interaction.

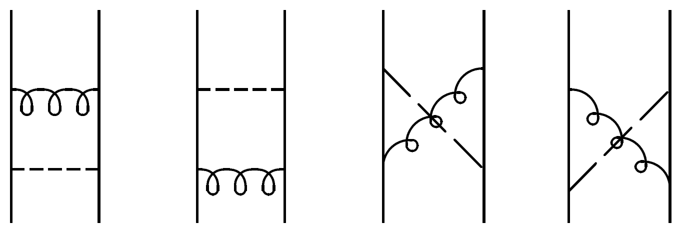


Figure 6.3: Contributions to the π-gluon exchange quark-quark scattering amplitude.

The determination of $U_{\pi G}$ starts with the calculation of the covariant field-theoretical amplitude $M_{\pi G}^{(4)}$ which can be expressed in terms of a scalar amplitude $s_j(s,t,u)$ and the spin invariants S_j, $j = 1, \ldots, 5$, as

$$M_{\pi G}^{(4)} = \sum_{j=1}^{5} s_j(s,t,u) S_j \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \ , \tag{6.16}$$

where s, t and u are Mandelstam variables related by $s+t+u=4m^2$ and defined as $s=-(p_1+p_2)^2$, $t=-(p_1'-p_1)^2$, $u=-(p_1'-p_2)^2$ (invariant momentum transfer variables). The spin invariants are defined as [90]

$$
\begin{aligned}
S_1 = \gamma_5^{(1)}\gamma_5^{(2)}(\gamma^{(1)}\cdot\gamma^{(2)}) \;, &\qquad S_2 = \{(\gamma^{(1)}\cdot P_1)(\gamma^{(2)}\cdot P_2), S_1\} \;, \\
S_3 = i[\gamma^{(1)}\cdot P_2 + \gamma^{(2)}\cdot P_1]S_1 \;, &\qquad S_4 = \gamma^{(1)}\cdot\gamma^{(2)}S_1 \;, \\
S_5 = \gamma_5^{(1)}\gamma_5^{(2)} \;, &
\end{aligned}
\tag{6.17}
$$

with $P_1=(p_1+p_1')$, $P_2=(p_2+p_2')$. These spin invariants can be related to the Fermi spin invariants ($SVTAP$) as [91]

$$
\begin{aligned}
S_1 = -A \;, &\qquad S_2 = \frac{1}{2}(s-u)(T-P) - \frac{1}{2}tS \;, \\
S_3 = \frac{s-u}{2m}(S+P) - 2mV + \frac{1}{2}(8m^2-t)T \;, & \\
S_4 = -2T + 4P \;, &\qquad S_5 = P \;.
\end{aligned}
\tag{6.18}
$$

The potential iteration term $J^{(4)} = K_\pi^{(2)}\tilde{g}K_G^{(2)} + K_G^{(2)}\tilde{g}K_\pi^{(2)}$ from the iterated π and gluon exchange is calculated with inclusion of the screening function for the gluon interaction $g(k^2)$ and the cut-off function $f(k^2)$ for the pion interaction mentioned in Section 6.1. One then has for the one-pion interaction

$$
K_\pi^{(2)} = \frac{f_{\pi qq}^2}{m_\pi^2}\cdot\frac{\gamma^{(1)}\cdot k\gamma_5^{(1)}\gamma^{(2)}\cdot k\gamma_5^{(2)}}{m_\pi^2+k^2}f(k^2)\boldsymbol{\tau}^{(2)}\cdot\boldsymbol{\tau}^{(2)} \;,
\tag{6.19}
$$

where the pseudovector pion-quark coupling constant $f_{\pi qq}$ can be calculated by relating it to the pion-nucleon coupling constant $f_{\pi NN}$, and the one-gluon interaction is

$$
K_G = -\frac{8\pi}{3}\alpha_S\frac{\gamma^{(1)}\cdot\gamma^{(2)}}{k^2}g(k^2) \;.
\tag{6.20}
$$

In Paper III the parametrization for $f(k^2)$ is the monopole form mentioned in Section 6.1, with $\Lambda_\chi = 1.0$ GeV, and for $\alpha_S g(k^2)$ the parametrization $\alpha_B(k^2)$ in Figure 6.1 is used. The resulting term $J^{(4)}$ can now be written in terms of five spin amplitudes R_i, $i=1,\ldots,5$, as

$$
J^{(4)} = \sum_{i=1}^{5} j_i(t,p^2)R_i\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)} \;,
\tag{6.21}
$$

where [90]

$$
\begin{aligned}
R_1 = \gamma_5^{(1)}\gamma_5^{(2)}(\gamma^{(1)}\cdot\gamma^{(2)}) &\qquad R_2 = \gamma_4^{(1)}\gamma_4^{(2)}R_1 \;, \\
R_3 = \{(\gamma_4^{(1)}+\gamma^{(2)}), R_1\} \;, &\qquad R_4\gamma^{(1)}\cdot\gamma^{(2)}R_1 \;,
\end{aligned}
$$

$$R_5 = \gamma_5^{(1)}\gamma_5^{(2)} \ . \tag{6.22}$$

To be able to subtract this term from the covariant amplitude the spin invariants S_j in the expression for $M_{\pi G}^{(4)}$ are re-expressed in terms of R_i and the scalar amplitudes $s_j(s,t,u)$ are transformed into amplitudes $r_j(s,t,u)$ related to the spin amplitudes R_i resulting in the potential

$$\begin{aligned} U_{\pi G} &= \sum_{i=1}^{5} R_i \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} [r_i(s,t,u) - j_i(t,p^2)] \\ &= \sum_{i=1}^{5} R_i \boldsymbol{\tau}^{(1)} \cdot \boldsymbol{\tau}^{(2)} \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\eta_i(s,t',u) - \bar{\eta}_i(t',s)}{t-t'}. \end{aligned} \tag{6.23}$$

In the last step of the above equation the scalar amplitudes have been calculated as dispersion relations, the imaginary (divergent) components of which cancel. This potential has a weak energy dependence and can be approximated as a local potential by using the adiabatic (non-relativistic) limit $\frac{p^2}{m} \to 0$ when the momentum transfer variable t is held fixed. One then makes a transformation to a two-component Pauli spinor representation using $\bar{u}(\mathbf{p}')^{(1)}\bar{u}(-\mathbf{p}')^{(2)} R_i u(\mathbf{p})^{(1)} u(-\mathbf{p})^{(2)} = \sum_\alpha X_{i\alpha}^{R\Omega} \tilde{\Omega}_\alpha$, where $X_{i\alpha}^{R\Omega}$ is a linear transformation, the elements of which are scalar functions of the energy and momentum transfer variables, and $\tilde{\Omega}_\alpha$ are five independent non-relativistic invariant operators in momentum space (cf. Eq. (5.14) for the non-relativistic reduction of the Fermi $SVTAP$ invariants). This results in a potential in momentum space of the form $< \mathbf{p}'|V|\mathbf{p} >= \sum_\alpha V_\alpha(t)\tilde{\Omega}_\alpha$. Fourier transformation into configuration space,

$$< \mathbf{r}'|V|\mathbf{r} >= \frac{1}{(2\pi)^6} \int \int d^3p d^3p' e^{i\mathbf{p}'\cdot\mathbf{r}'} < \mathbf{p}'|V|\mathbf{p} > e^{-i\mathbf{p}\cdot\mathbf{r}} \ , \tag{6.24}$$

where $< \mathbf{r}'|V|\mathbf{r} >= \delta(\mathbf{r}' - \mathbf{r}) V(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}^{(1)}, \boldsymbol{\sigma}^{(2)})$, results in a local potential in configuration space which can be decomposed as

$$V(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}^{(1)}, \boldsymbol{\sigma}^{(2)}) = \sum_\alpha V_\alpha(r)\Omega_\alpha \ . \tag{6.25}$$

The operators Ω_α are here defined as in Eq. (5.12). The components V_α, which depend only on $|\mathbf{r}| = r$, correspond to central (C), spin-orbit (SO), tensor (T), spin-spin (SS) and spin-orbit squared ($SO2$) components of the potential. The last ($SO2$) term gives small local contributions and is therefore discarded. The potential components contain dispersion integrals from some minimal value t_0 to infinity over the invariant momentum transfer t' (cf. Eq. (6.23)). Since in the non-relativistic limit the momentum is assumed to be small these integrals have to be cut off at high momenta. This is achieved partly by using an upper limit t_{max} for the integration and partly by the use of the pion cut-off and gluon screening functions mentioned in the beginning of this chapter.

To be able to estimate the strength of the different potential components one can compare them to the corresponding central and spin-orbit one-gluon exchange components and to the one-pion exchange components in the tensor and spin-spin cases. For (screened) one-gluon exchange one then assumes

$$V_C(OGE) = -\frac{2}{3}\alpha_S \frac{1}{r} e^{-r\Lambda_{QCD}} \ , \tag{6.26a}$$

$$V_{LS}(OGE) = \frac{\alpha_S}{m^2} \frac{(1 + r\Lambda_{QCD})}{r^3} e^{-r\Lambda_{QCD}} \ , \tag{6.26b}$$

and for one-pion exchange (with a monopole form factor form)

$$V_T(OPE) = \frac{g^2_{\pi qq}}{4\pi} \cdot \frac{m^3_\pi}{12m^2} \left\{ \frac{e^{-m_\pi r}}{m_\pi r} \left[1 + \frac{3}{m_\pi r} + \frac{3}{m^2_\pi r^2} \right] \right.$$
$$\left. - (\frac{\Lambda_\chi}{m_\pi})^3 \frac{e^{-\Lambda_\chi r}}{\Lambda_\chi r} \left[1 + \frac{3}{\Lambda_\chi r} + \frac{3}{\Lambda^2_\chi r^2} \right] \right\} \ , \tag{6.27a}$$

$$V_{SS}(OPE) = \frac{g^2_{\pi qq}}{4\pi} \cdot \frac{m^3_\pi}{12m^2} \left\{ \frac{e^{-m_\pi r}}{m_\pi r} - (\frac{\Lambda_\chi}{m_\pi})^3 \frac{e^{-\Lambda_\chi r}}{\Lambda_\chi r} \right\} \ . \tag{6.27b}$$

The results are shown in Figs. 2 - 5 in Paper III. The comparisons show that the central and spin-orbit components of the π-gluon exchange interaction have short range and are very weak for $r > 0.3$ fm. The tensor component of the π-gluon exchange interaction is of the same magnitude as the tensor component of the one-pion exchange interaction but with opposite sign, and thus the two contributions almost cancel, resulting in a very small flavor dependent tensor interaction between constituent quarks. The spin-spin components of the π-gluon and one-pion exchange interactions are, on the other hand, of the same order of magnitude but with the same sign and thus add. For very short distances the π-gluon exchange interaction is strongly dependent on the form for the pion and gluon interactions at high momentum.

7. Conclusions

In this thesis a model for effective quark-quark interactions between light and strange quarks in baryons has been studied and used for calculations of electromagnetic and weak observables in two different parametrizations [30, 31]. The so called chiral constituent quark model is based on the idea that the main features of the baryon spectrum can be explained by the introduction of a spin and flavor dependent hyperfine interaction [29] associated with the spontaneous breaking of chiral symmetry and the appearance of (approximate) Goldstone bosons (pseudoscalar mesons) that couple to constituent quarks in the region of low momentum transfer. The effective pseudoscalar meson exchange interaction is combined with a confining interaction that should describe the gross features of the spectrum. The chiral interaction approach is supported by e.g. $1/N_C$ expansion studies of $L = 1$ non-strange spectra [92] and a phenomenological analysis of the $L = 1$ negative parity spectra [93].

The chiral hyperfine interaction implies, due to requirements from the continuity equation, the presence of exchange magnetic moments. These tend to compensate the large underestimations of the baryon magnetic moments when including one-body relativistic corrections to the static magnetic moment operator. When using the parametrization of Ref. [30] for the chiral interaction potential a substantial part of the hyperfine interaction is due to some exchange mechanism other than effective pseudoscalar meson exchange and has to be treated phenomenologically. In the parametrization of Ref. [31], on the other hand, all of the hyperfine interaction can be interpreted as coming from effective pseudoscalar meson exchange. Both parametrizations give results that are in good agreement with the empirical proton and neutron magnetic moments, while only the parametrization of Ref. [30] was used to calculate magnetic moments for strange baryons, yielding values that also are in fairly good agreement with data.

The axial coupling constants that appear in weak semi-leptonic decays of light and strange baryons are overestimated in the static quark model. When including lowest-order relativistic corrections to these operators in the chiral constituent quark model these overestimations can be reduced. Under the assumption that the chiral interaction is caused by effective pseudoscalar meson exchange no exchange current contributions to the axial coupling constants appear, and it seems possible to, at least qualitatively, get a unified description of the axial coupling constants and the magnetic moments. The inclusion of the phenomenologically treated part of the chiral interaction of Ref. [30] could, in principle, give exchange current contributions to the axial coupling constants. These corrections have not, however, been calculated in this work, since more information on the phenomenological (short range) part of the interaction would then be required.

The charge radius of the nucleon is formed of two separate parts, the first coming from a charge distribution connected to the Dirac form factor and the second arising from the anomalous magnetic moment through the Pauli form factor. For the calculation of the Dirac part of the charge radius the exchange charge density operators corresponding to the Fermi-invariant decomposition of the quark-quark interaction have been constructed and their contributions along with one-body relativistic corrections to the static charge density operator were calculated in the parametrization of Ref. [31]. When combined with the "anomalous" part of the charge radius and contributions from an assumed constituent quark charge radius approximate agreement with empirical data was achieved. In the parametrization of Ref. [30] agreement with data seems to be possible only if the quarks are assumed to have considerably differing charge radii. This indicates that the parametrization of the chiral constituent quark model used in Ref. [31] is to be preferred, even though both parametrizations yield good baryon spectra and magnetic moments.

In the chiral constituent quark model the one-gluon exchange interaction of conventional constituent quark models is neglected. One reason for this comes from the results of phenomenological studies of the effective quark-gluon coupling strength α_S that suggest that α_S drops or freezes to a constant value [83, 84] at low momentum transfer. This seems to be supported by cooled lattice calculations [42] and calculations in the valence-QCD approximation [43, 44]. If in the low-energy regime there is a residual weak gluon exchange interaction along with the chiral pseudoscalar interaction (and the confining interaction) there will be an associated irreducible pseudoscalar meson-gluon exchange interaction. When calculating the strength of the different components of the π-gluon exchange part of this interaction a possible explanation for the absence of large spin-orbit splittings in the baryon spectrum can be obtained. The tensor component of the irreducible π-gluon exchange interaction in effect cancels a corresponding tensor part that should be included with the spin-spin part of the chiral interaction for states above the ground state. The spin-spin part of the π-gluon exchange interaction, on the other hand, adds to the chiral spin-spin interaction yielding a strong attractive flavor dependent spin-spin interaction which is needed in order to get the correct ordering of negative and positive parity states in the baryon spectrum.

In conclusion, the description of the effective quark-quark interaction in baryons seems to be a delicate problem. This thesis suggests that the chiral constituent quark model qualitatively seems to be a good candidate in describing both baryon spectra, magnetic moments, axial coupling constants and charge radii even though a fully relativistic treatment of these observables is clearly needed. By including a weak one-gluon exchange interaction and the associated irreducible π-gluon exchange interaction small spin-orbit splittings that appear in the chiral constituent quark model can be avoided, while the main (spin-spin) part of the hyperfine pseudoscalar interaction is enhanced. There could, of course, also be other exchange mechanisms, as e.g. two-pion exchange [89], that would contribute to the hyperfine interaction, and further study of these would be of interest.

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